Information Revelation and Bidding Behavior in Common Value English Auction

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Abstract

In the canonical common value English auction with asymmetric information, the identity of the insider is assumed to be mandatory known by the public and thus her existence often induces more aggressive competition. However, when the identity revelation is not enforceable, the insider can choose whether to reveal or not based on strategic interactions with her rivals. This could have no trivial impact on bidding outcomes: empirical evidence from Chinese Judicial Auction for real estate shows that the winning price declines when the insider voluntarily reveal her identity. To reconcile the conflicts between the data and theory, we introduce the information revelation decision process into the standard theoretical framework. Our model shows that the insider may reveal her identity when she receives a bad or very good signal. Also, the impact from the informed bidder decreases as the number of participants increases and the effect depends on the level of information frictions. Based on the theoretical framework, we overcome the identification challenge and construct parametric structure estimation algorithm to recover the value function distribution. Our estimation results find that first, common value part dominates the bidder's private value part of the selling house. Second, the noisy component is nontrivial. These two features give the informed bidder's opportunity to take advantage of strategic bidding. Informed bidder can leverage her information advantage to influence the bidding activities. For example, adopting both active and inactive bidding strategies, the insider can shift the winning price distribution leftward by dampening the bidding competition.

Keywords: Common Value, English auction, Information asymmetry, Information Revelation, Multiple equilibria, Chinese Judicial Auction

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1 Introduction

In an common value open ascending auction, bidders can learn rival's signals through their bidding activities, making themselves better by aggregating the information and avoiding winner's curse. But when a bidder has information advantage (insider or informed bidder)[1], she could influence bidding activities through strategic interaction. Often, an informed bidder has to pay more if she participates and wants to win the auction (Hernando-Veciana, 2009; Dionne et al., 2009). But the previous studies, both in theory and empirical, hold a key implicit assumption that the identity of the informed bidder is known to the public. **Things could change if the informed bidder can free to decide whether to reveal her identity or not.**

Nowadays, with the advancement IT technology, more and more auctions are holding through the internet under the open ascending format. One typical feature of these auction activities are such that bidders are anonymous and their identity is not revealed to the public. For example, to prevent the corruption and promote justice and transparent, more and more high valued and public asset disposal activities are processed through the online auction in China, e.g., Judicial auction, Land auction, etc. Since different agents have different levels of information towards the selling item, insiders in these auction can leverage her information advantage, especially the value of the identity, to influence the bidding activities. On the one hand, the informed bidder can hide her identity if hiding brings her higher expected revenue. On the other hand, the informed bidder may voluntarily reveal her identity to discourage the bidding competition or suppress information revelation process.

How does such behavior affect bidding outcomes still remains an open question. For example, using the data of Chinese Judaical auction, we plot distribution of weighted winning price for the auctions with and without identified informed bidder's participation in Figure 1, which illustrates the main data fact of this research:

Fact 1. *The existence of an insider (informed bidder) reduces the winning outcomes under open ascending auction.*

This data fact posts new challenge for classic auction theory, which shows that in open ascending auction, the insider induces more aggressive bidding activities. Understanding interactions between insiders and outsiders during the auction and, more importantly, revenue distribution between the auctioneers and bidders are crucial not only for economists, but also for organizers or institutions that aim to use auction for processing high valued assets. However, without carefully consideration of the

[1]Hereafter, we will use insider or informed bidder interchangeably. Moreover, for the convenience of referring, we assume insiders as she/her while normal bidders as he/his.

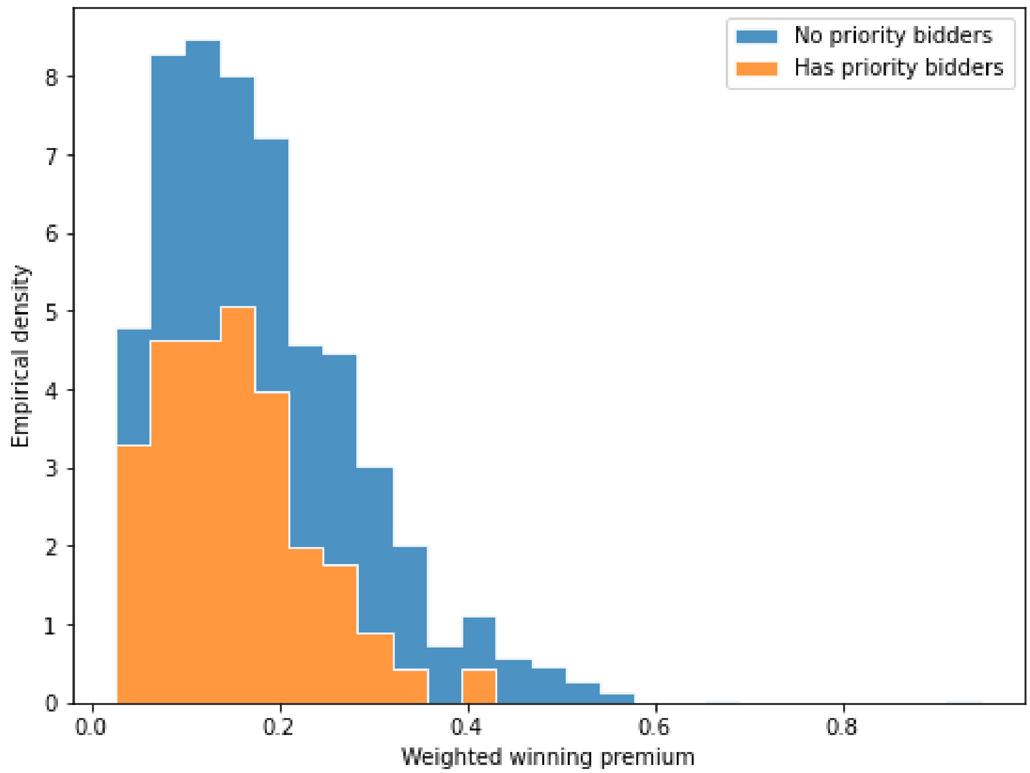

Figure 1: Statistics of the auctions for the insiders
This figure shows the weighted empirical density distribution of the winning premium under two different cases. *Auctions with an insider on average has lower weighted winning premium.*

information revelation process as well as the interactions between informed and uninformed bidders, the analysis could be biased as shown in Figure 1. In this research, we aim to reconcile the gap between the theory and data, and investigate the mechanism and decision rules behind the behavior of different groups of bidders. Moreover, by constructing a structure empirical auction model, we can further exercise different kinds of policy experiments that provides the policy suggestions for auction design and welfare improvement.

Specifically, we build a two-step Bayesian Nash equilibrium model with identity revelation decisions for informed bidder. In the first step, the informed bidder (if exists) makes the identity revelation decision. In the second step, all the bidders start the bidding competition under open ascending format and win the item until no new bid submits. The identity revelation rule for the informed bidder relies on whether revelation can dampen the competition and alter the information updating process to benefit herself or not. Mathematically speaking, identity revelation rule results in two cutoff points for the informed bidder's signal: conditional on the common value component, informed bidder will reveal her identity to the public if her signal either low or very high. Between the two cutoff points, the informed bidder will hide her identity for participation.

We further combine the model with detailed Chinese Judicial Auction housing data collected from major cities in China. One key features in the data is the existence of "priority bidder", which is defined as the person who has some inside information to the selling item, i.e., the insider. Examples are like the tenants of the house for sale or creditor of the asset. More importantly, the priority bidder can choose to self-identify themselves or not. With the help of the auction data, we can study the bidder's bidding behavior under different information structure. Moreover, we are able to answer how bidders leverage their asymmetric information for bidding strategies and shed light on the information premium allocation among bidders.

Empirically, studying these questions need to back out the bidder's value distribution first. In an ideal situation of an auction model, we bridge the bidding distribution with the value distribution under one to one mapping (Milgrom and Weber, 1982). But as emphasized in Bikhchandani et al. (2002), the equilibrium bidding strategy in the English auction involves the multiple equilibria problem. **This is because bidding prices only indicate the lower bounds of a bidder's evaluation on the selling item**. One typical bidding price could result from different bidding strategies, reflecting a wide range of possible private signals. Moreover, Haile and Tamer (2003) point out that in English auction, prices may rise in jumps of varying sizes, and active bidding by the rivals may discourage bidders to bid close to their evaluation. Hence, it is difficult to observe every bidder's highest dropout prices or expected valuation. To overcome the model incompleteness, we construct a structure econometric framework based on maximum simulated likelihood method to estimate parameters. After backing out the estimate, we can further explore how the information premium is distributed among different types of bidders. Additionally, we can conduct related policy experiments to evaluate how bidders utilize their asymmetric information to shape their bidding strategies..

This work departs from the previous analysis in many ways. First, to overcome the multiple equilibria and incompleteness of the model, this paper extends Haile and Tamer (2003) approach to estimate the value distribution in common value English auction. Rather than selecting a unique equilibrium bidding strategy, we use the bidding history to construct lower and upper bounds on the bidders' private signals to encompass all the potential bidding strategies. Moreover, since the the second highest bidder's last posting price equals to the dropout price by model implication, we leverage the observation of winning bids to narrow down the structure parameter sets. As the bidding ladder (the minimal bidding increment required by the auctioneer) is close to zero, the estimated results converge to point estimation.

Second, our theoretical model provides rational explanation for the data facts that can not be explained by the previous canonical theory. Based on the model, we are able to construct the structure estimation framework and design the identification strategy for parameters of interest. In our model, the value of the selling item consist of common value part which is identical for all the bidders and private value part which is bidder specific. Unlike the informed bidder, the uninformed bidder receives a noisy signal of the item value instead of the precise value of the time. To separate each one of them, we utilize the second highest bidder's bidding price and bidding distance among bidders to identify the private as well as the noisy component. Moreover, the variation for the number of bidders in different auctions can help us pin down the common value component. And the difference between auctions with or without informed bidder can further help us pin down the noisy component.

Third, regarding the interactions among informed and uninformed bidders, the learning process further implies that the informed bidder can significantly affect the auction outcome by strategically choosing bidding behavior. For instance, an informed bidder can bid either more or less aggressively during the auction. When the informed bidder intentionally reduce her bidding activities. The final winning bid distribution is more right skewed compared with the baseline case. A more interesting case lies in the active bidding activities. If the informed bidder bids too frequently, other bidders may not have enough chances to fully reveal their private information. In other word, the last bidding prices of these bidders are too far away from their true valuation. Hence, the information blocking effect rather than competition effect dominates the learning process. And the simulation results also verify the right skewed winning bid distribution. Hence, sometimes knowing more may be not good for the information disadvantaged agents.

Using the Chinese Judicial Auction data to back out the bidder's value distribution, I find a nontrivial noisy component of the bidder's signal (same level as the bidder's private value component). Also, the common value part dominates both the private and noisy part. This indicates a stronger power for the informed bidder to "manipulate" the auction outcomes since the insider's information is more valuable. Consistent with the model prediction, the priority bidder in the data display either very inactive or very active bidding activities. In the counterfactual analysis, the inactive bidding strategy from the informed bidders can reduce the auction outcome around 10% compared with the symmetric case. While the active bidding strategy also reduce auction outcome at the same level. From the perspective of the auction designers, the auction revenue can be increased if we can either reduce the noisy information or regulate the insider's behavior during the Judicial auction process.

Related Literature. This research connects to several strands of literature in theoretical and empirical auction studies, focusing on information(identity) revelation under common value English auction format.

First, the bidder's information premium depends not only on their information advantage but also on the auction format. A key factor influencing information disclosure is the reduction of uncertainty (Benoit and Dubra, 2006). In dynamic auction scenarios, such as English auctions, bidders are required to repeatedly reveal their valuations, which can turn an insider's advantage into a disadvantage (Hernando-Veciana and Tröge). This dynamic encourages greater competition and improved information aggregation. Conversely, in static (sealed-bid) auctions, empirical evidence from Hendricks and Porter (1988) shows that the strong winner's curse associated with better-informed bidders can negatively impact drainage lease prices, discouraging uninformed bidders from participating actively.

Furthermore, information disclosure often introduces challenges related to interdependent value distribution. Adding a common value component increases the model's complexity by introducing an additional signal dimension, complicating the mapping from value distribution to bidding strategies. Goeree and Offerman (2003) systematically analyze how different value sources influence efficiency and revenue, recommending that reducing uncertainty about the common value component can enhance efficiency. Tan argues that while disclosing private signals may erode bidders' competitive advantages, revealing common-value signals can improve social welfare and benefit sellers by fostering efficient bidding equilibria. In an experimental study, Grosskopf et al. (2018) observe behavioral divergences in auctions with partial identity disclosure: informed bidders often overbid due to overconfidence in their signals, while uninformed bidders strategically underbid to mitigate risks associated with adverse selection. Bobkova (2024) extends these ideas by emphasizing the critical importance of identifying which information components matter most. Thus, selecting an appropriate auction or information revelation mechanism is crucial. However, addressing multi-dimensional signals presents significant challenges. Fang and Morris (2006) and Jackson (2009) explore equilibrium-related issues in multi-dimensional settings, shedding light on the complexities involved in achieving optimal auction design.

Research on information disclosure often focuses on the intensive margin—how precisely bidders know opponents' signals. McClellan (2023) and Cao, Ma, and Xiang (2025) shift the focus to bidder identity in static auctions. Distinctly, this paper connects models to data, explaining bidding strategies in dynamic auctions under identity revelation. Using a structural econometric model, we estimate the information premium under varying conditions, facilitating welfare evaluations for auction design.

Identification is central in empirical auction research (Athey and Haile, 2002; Athey et al., 2011; Hendricks and Porter, 2007). Haile and Tamer (2003) address model incompleteness in English auctions, proposing bounds for value distribution in IPV settings. Athey and Haile (2002) guide auction format selection, while Athey et al. (2011) compare efficiency and revenue in English versus sealed-bid timber auctions. Testing the common value empirically remains difficult due to unobserved heterogeneity. Haile and Kitamura (2019) address this through a test for common values in first-price auctions. Additionally, Compiani et al. (2020) analyze U.S. offshore oil lease auctions, revealing that unobserved heterogeneity and identity-driven entry decisions significantly affect the auction's competitive structure.

Challenges such as multiple equilibria (Bikhchandani et al., 2002) and partial identification complicate auction analysis. Tang (2011) partially identifies revenue in sealed-bid auctions with affiliated signals. Chesher and Rosen (2017) use generalized instrumental variables for sharp identification. Komarova (2013) examines valuation distribution identification under varied data conditions. Aradillas-López et al. (2013) directly identify profits in ascending auctions without relying on valuations. Coey et al. (2017) extend bounds to asymmetric valuations and refine them when bidder identities are observed. With improved data, partial identification relaxes assumptions and enables robust inferences (Tamer, 2010). Recent advancements by Kline et al. (2021) and Kline and Tamer (2023) provide new tools for inference in partially identified models with applications in industrial organization.

This paper's empirical method builds on Haile and Tamer (2003) and structural models by Hong and Shum (2003), Aradillas-López et al. (2013), and Coey et al. (2017). To leverage data features, we employ maximum simulated likelihood estimation for structural estimation.

The rest of the paper is organized as follows. Section 2 introduce the institutional background for the data and provides the empirical evidence regarding the informational asymmetry problem. Section 3 builds a simple two-stage auction model that characterizes the insider's information revelation decision as well as the bidding behavior for different types of bidders. Based on the theoretical results, section 4 discusses identification strategies and construct the structure estimation procedure. Section 5 collects main estimation results and discuss the value of the information in different scenarios. In section 6, we further conduct a series of policy experiments. Finally, Section 7 concludes.

2 Institutional Background and Data Statistics

2.1 Online Chinese Judicial Auction

The primary objective of the Chinese Judicial Auction is to process confiscated or public assets or property rights equally, transparently and efficiently. In general, the local courts need to deal with 600 billion RMB or more worth of assets and property rights every year. The item for selling includes furniture, car, house, land, and etc. In this research, our focus is the house auction. Before the auction begins, the auctioneer (i.e., local court) will publish the announcement 15 to 45[2] days earlier and open the registration window for potential bidders. The announcement includes important information such as market evaluation, reservation price, the minimum bidding increment, location, the existence of the priority bidder, usage information, photos, documentation, plus the third-party evaluation report. Anyone who wants to attend can register on the platform with the required margins during the pre-auction period.[3] The margin will be refunded if the bidder fails to win the auction. If the item fails to be sold out at first time, the item has an extra chance to organize a second-chance auction again. So each item will have two chances to hold a normal auction. In the first-time auction, the reservation price should be no less than 80% (70% since 2017) of the evaluation rice of the item. In the second-chance auction, the reserve price can be no less than 80% (70% since 2017) of the first-time auction's reserve price.[4] If the item can not be sold successfully after the second-time auction, the item will turn to other disposal methods.

The online platform provides detailed information for each auction: the description of the item, the date for the auction, the winner of the auction, the reservation price, the evaluation price, the location of the item if the item was a real estate, the existence of a priority agent, the local court who organized the auction, the number of bidders in the auction, the length of bidding period. More importantly, we can observe the bidding history per auction.[5]

The auction is organized as an open ascending auction such that each bidder publicly submit bid against each other. The number of participated bidders is publicly known at the beginning of the auction. The auction begins at the reservation price of the selling item, i.e., a confiscated house. During the auction, the bidding ID for anyone who submits the bid successfully at the current bidding period will be announced to the public. And his/her bid is the current posting biding price, indicating the temporary highest bidding price. Whoever wants to win the item has to submit a bid no less than the posting price plus the integral multiple of minimum bidding increment, which can range from 5,000 to 50,000 RMB or more, depending on the house value. The auction ends until no new bid submitted and the item belongs to the highest posting bidder and the winner finish the final payment.

[2]check the date

[3]The required margin should be in the range of 5% to 20% of the item value. Usually, the local court sets the required margin as 10% of the item value. Since houses in China are high-valued asset, the platform also cooperates with banks to offer loan service.

[4]If the the reverse price in the first time, the auction is set to 80% (70%). In the second time, the auctioneer can set the reserve price no less than 64% (49%) of the item's evaluation value. In this research, our focus is on the fully active binding cases which we can relax the reserve price restrictions.

[5]The supreme court authorizes 7 platforms to do the Judicial auction. One of the platform dominates the market (90%), and our data come from this particular platform.

After the auction, the website will publish the winning bidder's information if the auction is sold successfully.

In this auction, bidders have heterogeneous evaluation towards the selling house. On the one side, bidders have their private utility to the location, house design, environment, etc. On the other side, the house also has the market value and also becomes the major financial asset for households. Among bidders, there is one typical type of bidder tagged by the platform called "priority bidder", who by definition has the correlation with the selling property which is equivalent to the insider or informed bidder of the auction. For example, the priority bidder could be the tenant or the landlord of the house for selling, or the shareholders of the company, etc.[6] At the very beginning, the bidder's identity is anonymous. The priority bidder (insider) can decide to report her identity to the platform, which will be announced to the public. Or she can choose to hide her identity and participate the auction as other normal bidders. Being the priority bidder enjoy the privilege to win the item when there is a tie and can replace the current posting price without adding extra increment bid. Unlike the previous definition of an insider, The insider here can free to decide whether to reveal her identity or not, which changes the interaction among different types of bidders in the auction.

In this research, I focus on the first-time housing auction not binding with the reserve price and the data sample ranges from 2015Q1 to 2019Q3. The data consists of representative cities[7] in the eastern and southern area of China with around 19,000 cleaned successful first-time house auctions.

2.2 Data Statistics

The data sample excludes auctions in which the bidder wins the auction at the reservation price. From the summary statistics in Table 1, we see that of all 19,295 auction observations, only 208 auctions have identified the priority bidders[8], which is only 1% of the total data sample. The number of bidders in most auctions is less than 10, illustrated in Figure 4. Moreover, we also see that around 10% of total auctions only have two bidders involved. The relative low participation rate implies that online Judicial Auction is not very public well-known and incurs non-trivial information searching frictions. The reserve ratio, which is defined by the reservation price over the evaluation price, ranges from 0.7 to 1, satisfying the reservation price regulation on the first-time auction. The winning

[6]The details of the regulation on priority bidder can be found on the platform.

[7]The selected cities include: Beijing, Shanghai, Tianjin, Nanjing, Suzhou, Xuzhou, Wuxi, Ningbo, Hangzhou, Wenzhou, Xiamen, Fuzhou, Guangzhou, Quanzhou, Shenzhen, Zhongshan, Dongguan covering 7 provinces distributed in the the eastern and southern area of China.

[8]Due to the technical issue, some auctions announce the existence of a priority bidder but do not reveal the identity during the bidding process. There are 232 observations for these auctions.

premium, defined as $\frac{\text{winning price}}{\text{evaluation price}}$, has a mean value equal to 1.061 with a relatively high volatile standard deviation 0.199. The positive premium indicates that Judaical auction helps to dispose confiscated assets in a fair market value though with high volatility. Moreover, the winning premium has a fat tail pattern: some auctions winning price can be more than 80% of the evaluation price. In the classical English auction, the bidding ladder, i.e., the minimum bidding increment required in the auction, will affect the efficiency of information revelation. Although the bidding increment is large in terms of absolute value (more than 10,000 RMB), compared to the total values of the item, the minimum bidding ladder is on average less than 0.4% of the reserve price (ladder ratio in the table). This implies that bidding ladder is not a serious issue in the analysis of this auction environment.

Table 1: Summary statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Median	Pctl(75)	Max
Total Priority bidders	440	-	-	-	-	-	-	
Identified	197	-	-	-	-	-	-	
Winning premium	19,295	1.061	0.199	0.761	0.915	1.016	1.166	1.819
Number of bidders	19,295	6.591	2.908	2	4	6	8	15
Reserve ratio	19,295	0.758	0.093	0.700	0.700	0.704	0.800	1.000
bidding Length	19,295	52.918	33.806	6	29	44	68	219
Ladder ratio	19,295	0.006	0.004	0.001	0.003	0.004	0.007	0.056

Besides the above mentioned statistics, there also exists salient heterogeneity across auctions in terms of the location, facility, house area, etc. Figure 5 shows how winning premium is distributed under different properties (housing area, evaluation price per square meters, geographic location)[9]. In Figure 5a, we see that the larger the house area is, the more positive skewed the distribution will be. Also, house area in the third quartile (between 122 $m^2 \sim 163$ m^2) is highly right skewed, i.e., most often discounted. The price gap between the market price and winning price implies that participating in the judicial auction, insiders can take advantage of the gap.

When classified by the evaluation price per square meters in Figure 5b, the winning premium for the houses in the most expensive quartile is the one with the most positive skewed distribution. Specifically, the higher the price per square meters on the market, the more severe discount during the auction. On the contrary, houses with cheaper price per square meters have a fat tile that a non-trivial share of these house have very high winning premium (>1.5). Such difference provides another piece of evidence showing the information frictions on the Judaical auction activities. Besides, The heterogeneity among judicial auctions are also reflected in the geographic locations. Figure 5c

[9] In figure 5, I do not differentiate the auctions with or without priority bidders. Here I want to capture the general data patterns for Online Judicial Auction.

displays the skewness of the winning premium varies across different provinces in China, indicating the necessary to control for the geographic differences when conducting the empirical analysis. Among these provinces and Municipalities, auctions in Beijing, Tianjin and Guangdong have the longer tails.

In canonical auction theory, when the identity of an informed bidder (i.e., priority bidder) is known to the public, it can induce more aggressive bidding activities during the auction (Hernando-Veciana, 2009). But under a small group of number of bidders, the insider may leverage her information advantages to affect other bidders' behavior. In the next subsection, we utilize econometric tools to analyze the impact of the priority bidder on the bidding activities.

2.3 Priority Bidders and Information Asymmetry

In Judicial auction, being a priority bidder, i.e., revealing her insider's identity to the public, is not a mandatory requirement. Comparing to a more aggressive competition scenario, saving one extra bidding increment does not seem to have enough incentive for insiders to voluntarily reveal her identity. However, in auctions with the priority bidder, we indeed observe that the priority bidder bids actively and many of them finally win the item. Hence, there must exist alternative incentives that encourage the insider to reveal their identity. To disentangle the difference between auctions with and without priority bidder, we first separate out the two groups of auction. From the number of bidders in Figure **??**, we see auctions with priority bidder on average have fewer participants under open ascending Judicial Auction. Figure 1 shows that the winning premium weighted by bidding activities for auctions with the priority bidders has lower level value distribution than those without priority bidder, which provides our main data fact.

To further specify the effect of insider's participation on the bidding outcomes, we conduct the following econometric analysis:

$$y_{\tau,t} = \delta 1_{\tau,t}\{priority\} + \psi W_{\tau,t} + \phi Z_{\tau,t} + \mu_g + \lambda_t + \varepsilon_{\tau,t},$$

where the $y_{\tau,t}$ indicates the dependent variable: winning premium, defined as wining price over the evaluation price for the selling item; the indicator $1_{\tau,t}\{priority\}$ represents whether the current auction τ has a priority bidder or not;[10] $W_{\tau,t}$ includes the bidding activities and number of bidders; $Z_{\tau,t}$ represents the control variables, such as the reserve price, the housing area, etc.; μ_g and λ_t

[10]In the data, some priority bidders' identity is revealed to the public while in certain auctions, the platform only notified the existence of an informed bidder but did not announce the bidder's identity. Hence, we have two sub-groups for the priority bidder in the econometric analysis.

indicates the fixed effect for cities and years, respectively; and $\varepsilon_{\tau,t}$ is the error terms assuming independent of other covariates. The key interest variable here is δ, representing the effect of the existence of the priority bidder on the winning outcome. Moreover, ψ shows how the bidding activities affect the final outcomes.

Since the data have both identified insiders and unidentified insiders, we can separate these insiders into two subgroups and check the impacts on the winning premium. Table 2 lists the regression results. Column 1 and 2 shows the effect of the priority bidder on the winning premium where the identity of the insiders is known to the public. While column 3 and 4 shows the results for those unidentified insiders, i.e., other bidders does not know who is the which bid comes from the priority bidder. In the theory, researchers predict that the existence of an insider under an open ascending common value auction should encourage more aggressive bidding activities, which lead to a higher bidding outcomes. However, the regression indicates the opposite effect: the existence of an identified priority bidder will cause the reduction winning premium. The coefficients are both statistically and economically significant. Other covariates, such as number of bidders and length of bidding activities display positive and significant coefficients. The negative impact from the identified insider challenge the classical theory prediction. The contrasting regression results between identified and unidentified priority bidders illustrate the importance of the insider's identity, which is abstract away in the previous literature.

Moreover, in the data, we also notice that priority bidders have quite heterogeneous bidding behavior: someone bid very actively and finally win the auction, while others behave very inactively. The heterogeneous priority bidder's bidding behavior is one of the distinctive features in the data, implying the evidence of the strategic behavior among informed bidders. To explain the negative impact from the identified insider, one possible reason is that the identified priority bidder in these auctions bid very inactively. These identified insiders have lower individual valuation for the item. Seeing the early dropout for the priority bidder, other bidders will bid more cautiously. However, data show that about three quarters of these identified insiders bid very actively, either win the item or be the second highest bidder of the auction.

Regarding the bidding activity, we classify these identified insider bidders into active and inactive sub-group. Insiders that rank top two of the auction belong to the active group, while the rest of bidders are marked as inactive. Hence we get 126 active and 83 inactive insiders, respectively. Considering the huge sample size difference between auctions with and without priority bidders, I employ the nearest matching techniques to match the treatment group and control group (auctions without priority bidders).[11] Table 3 displays these findings.

[11] I employ the nearest matching method where the related metrics include the number of bidders, whether the auction

Table 2: Impact from identified and unidentified insiders on bidding outcomes (Pooled)

	Winning premium			
	(1)	(2)	(3)	(4)
Has insider	-0.021^{**}	-0.033^{***}	0.022^{***}	0.023^{***}
	(0.009)	(0.009)	(0.008)	(0.008)
# of bidders	0.021^{***}	0.020^{***}	0.021^{***}	0.020^{***}
	(0.0004)	(0.0004)	(0.0004)	(0.0004)
Bidding length (log)	0.110^{***}	0.116^{***}	0.110^{***}	0.116^{***}
	(0.002)	(0.002)	(0.002)	(0.002)
Bidding spread (log)	-0.007^{***}	-0.007^{***}	-0.008^{***}	-0.008^{***}
	(0.001)	(0.001)	(0.001)	(0.001)
Reserve ratio	1.143^{***}	1.177^{***}	1.143^{***}	1.176^{***}
	(0.010)	(0.012)	(0.010)	(0.012)
Jump bid	0.069^{***}	0.074^{***}	0.070^{***}	0.074^{***}
	(0.002)	(0.002)	(0.002)	(0.002)
Price per m^2 (log)	-0.082^{***}	-0.090^{***}	-0.082^{***}	-0.090^{***}
	(0.001)	(0.001)	(0.001)	(0.001)
House area (log)	-0.065^{***}	-0.068^{***}	-0.065^{***}	-0.067^{***}
	(0.001)	(0.001)	(0.001)	(0.001)
Model	Identified priority bidders		Unidentified priority bidders	
Fixed effect	No	Yes	No	Yes
Observations	19,052	19,052	19,098	19,098
Observations for insiders	197	197	243	243
R^2	0.605	0.637	0.606	0.638
Adjusted R^2	0.604	0.637	0.605	0.637
Residual Std. Error	0.125	0.120	0.124	0.119
	(df = 19043)	(df = 19022)	(df = 19089)	(df = 19068)
F Statistic	$3,640.099^{***}$	$1,151.813^{***}$	$3,662.467^{***}$	$1,158.450^{***}$
	(df = 8; 19043)	(df = 29; 19022)	(df = 8; 19089)	(df = 29; 19068)

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 3: Impact from active, inactive and unidentified siders (Matched)

	Winning premium					
	(1)	(2)	(3)	(4)	(5)	(6)
Has insider	-0.060^{***}	-0.064^{***}	-0.019	-0.034	0.013	0.014^{*}
	(0.019)	(0.018)	(0.027)	(0.025)	(0.009)	(0.009)
# of bidders	0.023^{***}	0.019^{***}	0.026^{***}	0.024^{***}	0.017^{***}	0.018^{***}
	(0.003)	(0.003)	(0.004)	(0.004)	(0.002)	(0.002)
Bidding length (log)	0.144^{***}	0.181^{***}	0.261^{***}	0.240^{***}	0.114^{***}	0.117^{***}
	(0.017)	(0.018)	(0.028)	(0.028)	(0.012)	(0.012)
Bidding spread (log)	-0.003	-0.006	-0.030^{***}	-0.015^{**}	-0.005	-0.004
	(0.005)	(0.005)	(0.008)	(0.007)	(0.004)	(0.003)
Reserve ratio	1.074^{***}	1.149^{***}	1.266^{***}	1.234^{***}	1.166^{***}	1.194^{***}
	(0.073)	(0.077)	(0.122)	(0.154)	(0.039)	(0.043)
Jump bid	0.025	0.047^{***}	0.023	0.049^{**}	0.086^{***}	0.084^{***}
	(0.017)	(0.017)	(0.025)	(0.023)	(0.010)	(0.009)
Price per m^2	-0.113^{***}	-0.086^{***}	-0.117^{***}	-0.143^{***}	-0.094^{***}	-0.103^{***}
	(0.009)	(0.013)	(0.014)	(0.034)	(0.006)	(0.008)
House area (log)	-0.123^{***}	-0.136^{***}	-0.155^{***}	-0.153^{***}	-0.090^{***}	-0.061^{***}
	(0.008)	(0.013)	(0.014)	(0.021)	(0.006)	(0.007)
Model	Active priority bidders		Inactive priority bidders		Unidentified priority bidders	
Observations	412	412	200	200	720	720
Fixed effect	No	Yes	No	Yes	No	Yes
R^2	0.638	0.730	0.738	0.768	0.696	0.748
Adjusted R^2	0.631	0.712	0.727	0.744	0.692	0.738
Residual Std. Error	0.120 (df = 403)	0.108 (df = 386)	0.121 (df = 191)	0.108 (df = 180)	0.105 (df = 711)	0.098 (df = 693)
F Statistic	88.930^{***}	41.737^{***}	67.272^{***}	31.410^{***}	203.153^{***}	79.003^{***}
	(df = 8; 403)	(df = 25; 386)	(df = 8; 191)	(df = 19; 180)	(df = 8; 711)	(df = 26; 693)

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Column 1 and 2 list the matched active insider's regression outcome, column 3 and 4 list the inactive ones, and column 5 and 6 list the unidentified priority bidder's results. The odd columns ignore the fixed effect while the even columns control the cities and year's fixed effect. The table shows several distinct features. First, the negatives significance holds for the active group. In column 2, the existence of an active priority bidder (δ) is significantly negative. Conversely, we do not find significance from the inactive group (Column 3 and 4). One possible explanation is that an inactive insider does not have any contribution to the information learning process. When checking the matched outcomes from the unidentified group, it is consistently positive and significant. This reinforce the previous conjecture that when the identity of an informed bidder is unknown other bidders will bid more aggressively. We further conduct a series of robustness check which all support the main results.

From above reduced form analysis, we see that when an insider reveal her identity to the public, the final outcomes will be lower than the case without the priority bidder. *This consolidate an important debate about the value of the information, knowing more information may be harmful.* To better understanding the mechanism behind the data evidence, we introduce the Bayesian beliefs and updating process into the canonical model aiming to reconcile the conflicts between data and classical auction theory. The details will be discussed in details in the next section.

3 Model

Regarding the conflicts between the theory and data evidence, the paper considers a model of two-stage common value English auction. In the first stage, the insider will make identity (information) revelation decision based on her received signal. And in the second stage, all other bidders receive their signals and based on the first stage information revelation decisions, bidders start bidding competition. For model traceability and identification feasibility (Athey and Haile, 2002), we assume a parameter settings for our model. In the following subsections, we first defined the model environment. Then we characterize the the insider's information revelation decisions and discuss the strategic bidding behavior between informed and uninformed bidders. Finally, we derive the equilibrium bidding strategies under the non-compulsory information revelation environment.

has jump bid or not, the reserve ratio, the bidding length, the price per square meters, the housing areas and the cities and years. Then we get the matched control group for the whole priority bidders, the active subgroup and the inactive subgroup. For robustness test, I also apply the balance-sample size frontier matching method developed by King, Lucas, and Nielsen (2017). And the results are even more salient.

Table 4: Impact from active, inactive and unidentified siders (Robustness test)

	Winning premium					
	(1)	(2)	(3)	(4)	(5)	(6)
Has priority bidder	−0.071**	−0.075**	0.034	0.025	0.015	0.023**
	(0.033)	(0.031)	(0.040)	(0.038)	(0.011)	(0.011)
# of bidders	0.021***	0.028***	0.023***	0.021***	0.016***	0.015***
	(0.005)	(0.005)	(0.007)	(0.008)	(0.002)	(0.003)
Bidding length (log)	0.114***	0.148***	0.303***	0.297***	0.112***	0.117***
	(0.029)	(0.031)	(0.043)	(0.052)	(0.016)	(0.017)
Bidding spread (log)	0.011	−0.0003	−0.040***	−0.029*	−0.010**	−0.006
	(0.010)	(0.010)	(0.014)	(0.016)	(0.005)	(0.005)
Reserve ratio	1.096***	1.207***	1.365***	1.329***	1.135***	1.189***
	(0.128)	(0.156)	(0.169)	(0.257)	(0.052)	(0.070)
Jump bid	0.043	0.050	−0.035	−0.030	0.086***	0.101***
	(0.034)	(0.033)	(0.044)	(0.043)	(0.014)	(0.014)
Price per m^2	−0.106***	−0.110***	−0.102***	−0.093*	−0.090***	−0.092***
	(0.014)	(0.020)	(0.018)	(0.049)	(0.008)	(0.012)
House area (log)	−0.117***	−0.094***	−0.168***	−0.167***	−0.098***	−0.062***
	(0.012)	(0.018)	(0.019)	(0.028)	(0.008)	(0.011)
Model	Active priority bidders		Inactive priority bidders		Unidentified priority bidders	
Observations	199	196	94	96	342	344
Fixed effect	No	Yes	No	Yes	No	Yes
R^2	0.596	0.701	0.753	0.792	0.723	0.770
Adjusted R^2	0.579	0.657	0.730	0.739	0.716	0.751
Residual Std. Error	0.137 (df = 190)	0.126 (df = 170)	0.121 (df = 85)	0.119 (df = 76)	0.100 (df = 333)	0.097 (df = 317)
F Statistic	35.093***	15.940***	32.351***	15.186***	108.481***	40.876***
	(df = 8; 190)	(df = 25; 170)	(df = 8; 85)	(df = 19; 76)	(df = 8; 333)	(df = 26; 317)
Note:						*p<0.1; **p<0.05; ***p<0.01

3.1 Environment setup

In this environment, each auction τ is associated with observed characteristics $M_\tau \in \mathbb{M}$, which includes the reservation price γ, the minimum bidding increment Δ, and other public released information. Besides, there are some characteristics $U_\tau \in \mathbb{U}$ not known by econometricians. The risk neutral bidder i values the object at $V_{i,\tau}$, but only receives a private and noisy signal $X_{i,\tau}$ of the valuation $V_{i,\tau}$. Let $V_\tau = (V_{1,\tau},...,V_{N_\tau,\tau})$, $X_\tau = (X_{1,\tau},...,X_{N_\tau,\tau})$, and $X_{-i,\tau} = X_\tau \backslash X_{i,\tau}$. The number of bidders entering auction τ is denoted by N_τ. In the biding stage, the realizations of $\Omega_\tau = (M_\tau, U_\tau)$ are common knowledge among bidders, as well as the distribution of $(X_\tau, V_\tau)|\Omega_\tau$. And let $F_{XV}(X_\tau, V_\tau|N_\tau, \Omega_\tau)$ denote the joint distribution of signals and valuations conditional on (N_τ, Ω_τ) (Compiani et al., 2020). Here we introduce our first primitive assumption.

Assumption 1. *(i) $\forall\, n \in supp\, N_\tau|\omega$ and $\omega \in \Omega_\tau$, $F_{XV}(X_\tau, V_\tau|n,\omega)$ has a continuously differentiable joint density that is affiliated, exchangeable in the indices $n = 1,...,N_\tau$, and positive in the support of X and V; (ii) For each bidder i in auction τ, the joint distribution function $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau = n, \Omega_\tau = \omega]$ exists and is strictly increasing in $X_{i\tau}$.*

Similar to Compiani et al. (2020), Assumption 1 helps to propose an analytical and trackable solution method for the value and signal distribution. Let us ignore the subscript τ first and introduce the information structure. The value of the object to (uninformed) bidder i[12], V_i, is assumed to take a multiplicative form $V_i = A_i \cdot C$, where A_i is a private value for bidders and C is a common value component unknown to all bidders in the auction. In this model, C and A_i are assumed to be independent from each other and follow log normal distribution. For the noisy signal, we have $X_i = A_i \cdot C \cdot E_i$, where $E_i = e^{\sigma_\varepsilon \xi_i}$ and ξ_i is an (unobserved) error term that follows a standard normal distribution (i.e., $\xi \sim N(0,1)$). Within an auction, the difference here is that within an auction C is identical for all bidders, i.e., the common value component, while A_i and E_i is bidder's specific. Let $c \equiv \ln C$, $a_i \equiv \ln A_i$, and $\varepsilon_i = \ln E_i$, we have:

$$v_i = a_i + c. \tag{1}$$

where

$$c \sim N(\mu_c, \sigma_c^2),$$
$$a_i \sim N(0, \sigma_a^2).$$

Given such information structure, if we let $v_i \equiv \ln V_i$ and $x_i \equiv \ln X_i$, conditioning on M, the joint distribution of $(V_i, X_i, i = 1...,N) = \exp(v_i, x_i, i = 1,..,N)$ is fully characterized by the parameter set

[12] In this paper, we use uninformed bidder or outsider and informed bidder or insider, interchangeably.

$\theta = \{\mu_c, \sigma_c, \sigma_a, \sigma_\varepsilon\}$. In this paper, we adopt the convention that lower case letters denote logs and upper case letters levels hereafter.

The above discussion focuses on the standard outsider's information structure. If there is an insider (informed bidder) in the auction, $i = \mathfrak{I}$, she is assumed to be free of information friction and able to separate out a_I and c:

$$x_I = v_I = c + a_I \tag{2}$$

Compared with symmetric case, the information frictions comes from two sources. First, the extra noisy intervention block uninformed bidders from learning the true value of the item. Second, the higher uncertainty or imprecise prediction of market fluctuation for the common value part that discourage bidders for the competition. The two sources characterize both the extensive and intensive margin of the information frictions for bidders in an open ascending auction. Since most of auctions in the data have at most one insider and our focus is on the interaction between insider and outsiders, we have the second assumption about the number of insiders.

Assumption 2. *An auction has at most one insider and the insider knows precisely the common value component.*

Without loss of generality, bidders are indexed by $i = 1, ...N_\tau$ where the ordering $1, ..., N_\tau$ indicates the order of latest bidding price of each bidder, so that bidder N's highest bidding price is the lowest among all the bidders, and bidder $i = 1$ wins the auction. In a typical common value English auction, every new bid will reveal some information to the public and other bidders can update their evolution based on the new bidding price, i.e., learning process. Here we introduce the assumption that regulates bidding behavior à la Haile and Tamer (2003):

Assumption 3. *Bidders do not allow an opponent to win at a price that they are willing to bid and they will never bid higher than* $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, \Omega_\tau]$

Due to the bidding mechanism in this model, bidders may leave the auction with the highest bid way below his/her valuation, as described in Haile and Tamer (2003). To circumvent this incompleteness issue, we leverage **Assumption 3** to recover the lower and upper bounds of the rivals' signals from bidding prices to construct the equilibrium bidding strategies.

3.2 Insider's information revelation decisions

Under classic theoretical framework for common value open ascending auction, the existence of a high-valued informed bidder will encourage the bidding activities. This is because the dynamic bidding process will "force" bidders to reveal their signals through bidding prices, resulting in more aggressive competition and "insider's curse". However, besides the provision of more precise information to the public, the private and noisy components of uninformed bidders amplify the winner's curse, which gives the informed bidder's advantage to manipulate the market by leveraging the identity revelation decisions. Hence, depending which effect dominates, the informed bidder may or may not reveal her identity.

Since the insider carries more precise information about the common value part, uninformed bidders will put more weights on the informed bidder's inferred signals $x_I(p_I)$ while reducing the weights from other rivals. It is important to regulate outsiders' prior believes about the insider's arrival probability. Without loss of generality, if the arrival probability of an informed bidder is small enough, uninformed bidder can ignore it existence unless shown up when formulating their bidding strategies, which gives the following high-order assumption:

Assumption 4. *Uninformed bidders have a prior belief q towards the existence of an insider in the auction. Since q is small $q \to 0$, when formulating the bidding strategy, unless the insider shows up, uninformed bidders ignore the participation of the insider.*

This assumption greatly helps us reduce the complexity when comparing the uninformed bidder's expected valuation under symmetric and asymmetric cases without scarifying key features of the model. From the property of the normal distribution and conditional expectation, the constant term for the asymmetric environment is lower than the symmetric one.[13] Together with the coefficient of x_I in different scenarios, we get the crossing point, shown in Figure 2. From Figure 2, we see that, given the common value realization c, if the informed bidder's signal is less than the cross point ($x_I < x_{cut}$), she has the incentive to reveal identity. The results are summarized in lemma 1.

Lemma 1. *Under the positive mean value of the value distribution, $\mu > 0$, according to the updating rules of uninformed bidders, there exists a cutoff point x_{cut} for the informed bidder, such that below the cutoff point, the informed bidder will reveal her identity to the market. Also the cutoff point is increasing with common value c. Defining the function $c = g(x_{cut})$, we have*

$$c = g(x_{cut}) = \gamma_0 + \gamma_1 x_{cut} \tag{3}$$

[13] This holds when the mean value is positive of the value distribution $\mu > 0$.

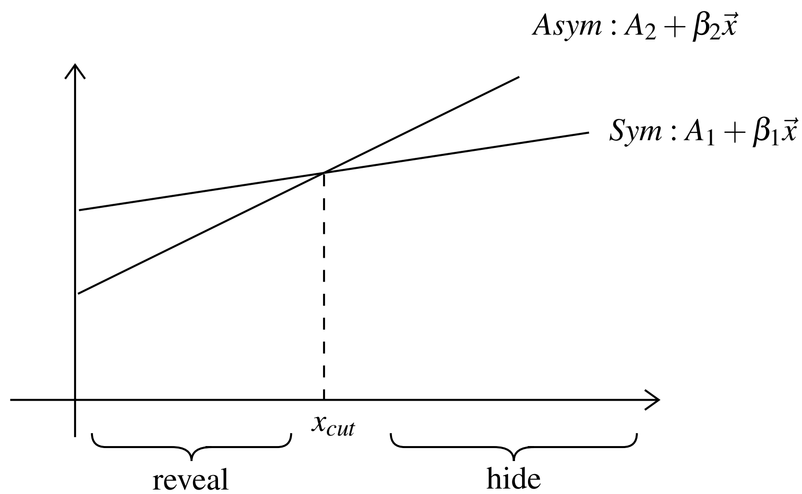

Figure 2: Information revelation decision and cutoff point

With $\gamma_0 < 0$ *and* $\gamma_1 > 1$.

The proof and details can be checked in the appendix A2.

Lemma 1 *characterizes the situation when the informed bidder's private value component is small.* As indicated by the coefficient of γ_1, when the common value realization is positive $c > 0$, the informed bidder will reveal her identity only if $0 < x_I < x_{cut} < c$, implying $a_I < 0$. However, under such scenario, the winning chances for the informed bidder is low because of the small private value component for the item. Here, a natural question rises: does the informed bidder also have an incentive to disclose her identity upon receiving a very high signal?

The answer is affirmative. This is because the informed bidder can leverage her information advantage to amplify the winner's curse effect so as to discourage other rivals bidding activities. Specifically, as bidding price increases, uninformed rivals are worrying that they may over estimate the true valuation of the selling item due to the noisy component of their signals. For example, when the uninformed bidder i perceive that the insider's signal is no less than his:

$$c + a_i + \varepsilon_i = x_i \leq x_I = c + a_I,$$
$$a_i + \varepsilon_i \leq a_I.$$

Unless the uninformed bidder is very confident that he gets a very high private component, i.e., $\varepsilon_i \leq 0$, it is very risky to continue bidding and win the auction. Such kind of winner's curse could push uninformed bidders to drop out early. In other words, they stop bidding when the informed bidder's signal exceeds their expected valuation, $\mathbb{E}[v_i|x_i]_{i=2} < x_I$. Taking the expectation of the

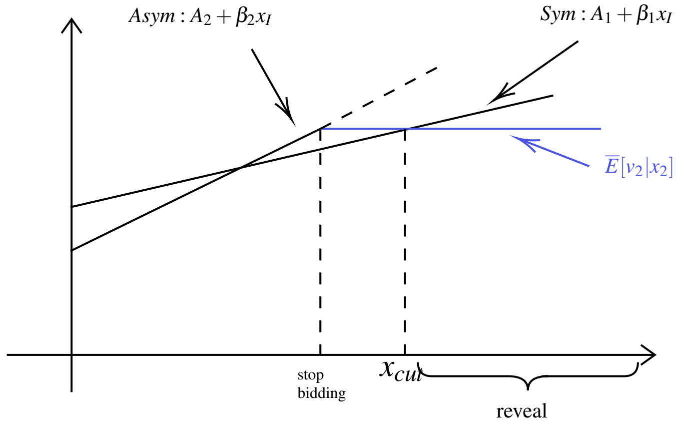

Figure 3: Information revelation decision and the second cutoff point

highest order statistics of $n-1$ uninformed bidders, $x_{cut} = \mathbb{E}[\mathbb{E}[v_i|x_i]_{1:n-1}]$, this constructs our second cutoff point for the informed bidder, as shown in Figure 3. The following lemma summarizes the construction of second cutoff point for insider's identity revelation decision.

Lemma 2. *Under the positive mean value of the value distribution, the participation of a high valued informed bidder can deceive uninformed rivals to stop bidding early. There exists a second cutoff point x_{cut} for the informed bidder such that above the cutoff point, the informed bidder will reveal her identity to the market. The cutoff point is determined by the first order statistics of the uninformed rivals' valuation.*

$$\begin{aligned} x_{cut} &= \mathbb{E}[\mathbb{E}[c + a_{1:n-1}|x_{1:n-1}]] \\ &= c + \mathbb{E}[\mathbb{E}[a_{1:n-1}|y_{1:n-1}]] \end{aligned} \tag{4}$$

where

$$y_{1:n-1} = (a+\varepsilon)_{y_{1:n-1}}$$

From Lemma 1 and 2, we have the whole picture of the informed bidder's identity revelation decision, which is summarized in the following proposition.

Proposition 1. *Based on Lemma 1 and 2, assuming the level of informed bidder's signal is x_I, she will*

1. reveal her identity to the public if $x_I < x^1_{cut}$ where x^1_{cut} is determined in condition (3);
2. hide her identity if $x^1_{cut} < x_I < x^2_{cut}$ where x^2_{cut} is determined in condition (4);
3. reveal her identity to the public if $x_I > x^2_{cut}$.

Because the informed bidder will truthfully reveal her identity, other uninformed bidders will predict the potential common value components and update their conditional expected valuation towards the selling item accordingly. The next subsection discusses the strategic interactions between informed and uninformed bidders' bidding behavior in detail.

3.3 Strategic bidding behavior between informed and uninformed bidders

Before the auction begin, depending on the informed bidder's information revelation decisions, the uninformed bidders learns the identity information. In the classic button auction, the uninformed bidders' bidding strategy follows the condition that bidders' conditional expected evaluation equals to their dropout prices. In our model, the equivalent no longer holds. Still, we know that the last bidding price of a bidder indicates the lower bound of her/his signal (Haile and Tamer, 2003), while the winning price automatically becomes the upper bound for all bidders but the winner. Hence, we can construct the system of inequalities for bidders.

To better approximate the real Judaical Auction activities while keep the model tractability. The auction is organized in the following mechanisms.

1. Whenever a bidder submits the first price, the auction begins denoted the period by $t = 1$. When the auction begins, each bidder can decide whether to submit the bid or not at any time.
2. At the beginning of period t,
   (a) bidders decide whether to submit new bid or not;
   (b) the auctioneer then collects all the submitted bids and post highest bidding price along with the corresponding bidder[14];
   (c) The information is revealed and the auction move to the next period $t+1$;
3. Based on the bidding history and the posting price, all bidders update their beliefs about their rivals' signals, make bidding decisions, and submit the new bids again.
4. If there is no more new bid, the posting bidder wins the item and the auction ends (T).

The bidding functions for bidder i in period t are defined as $\beta_i^t(X_i;\Omega_t),\ t = 1,\dots,T$[15] , where X_i denotes bidder i's private signal and Ω_t denotes the public information set (defined as before) at

[14]If there exists a tie, the auctioneer randomly picks one of them as the posting bidder.

[15]More rigoursly speaking, $\beta_i^t(X_i;\Omega_t)$ indicates the bidder i's highest possible bidding price that

period t. Specifically, Ω_t includes the bidding history up to period t, the common knowledge of M_τ, the number of bidders N_τ and the identity revelation of the insider. The function $\beta_i^t(X_i;\Omega_t)$ also indicate bidder i at which price she should stop bidding given the current information set Ω_t. Within the interval between the posting price P_{t-1} and $\beta_i^t(X_i;\Omega_t)$, bidder i can submit any price equal to $P_{t-1}+k\Delta, k\in\mathbb{N}_+$, where Δ represents the minimum bidding increment. The set of bidding functions $\{\beta_i^1(X_i;\Omega_t),..,\beta_i^T(X_i;\Omega_t)\}$ for bidders $i=1,...,N$ are common knowledge. Although we can not observe the dropout prices directly, the bidding history tells us the lower and upper bound that can help us to define the equilibrium bidding strategies and back out the value distribution (Haile and Tamer, 2003).

From period 1 to T, we can further slice into $n-1$ rounds like Hong and Shum (2003), indexed $k=0,...,n-2$. In round 0, all N bidders are presumed actively submitting the bidding price, and in round k, only $n-k$ bidders are virtually active: each round ends when a bidder no longer submits new bidding price. Then, we denote $\beta_i^k(X_i;\Omega_k)$ as bidder i's *pivotal bidding function in round k,* which reflects bidder i's *conditional expected valuation towards the selling property.* In the following subsections, we first examine the scenario in which the informed bidder's identity is revealed, followed by an analysis of the scenario where the identity remains concealed.

Equilibrium Bidding functions when the informed bidder reveals her identity In this scenario, the identity of the insider becomes the common knowledge. In the auction, uninformed bidders will update their expected valuation from other bidder's bidding activities. Particularly, they will put higher weights for the informed rival's signal implied by bidding price, to help them recover the value of the selling goods. While the insider knows the value of the selling item precisely, she does not rely on other rivals' signals.

For notational convenience, the informed bidder's bidding function is defined as $\beta_I^k(X_I,\Omega_k)$. In round k ($0\leq k\leq n-2$), fixing the realizations of $x_n,...,x_{n-k+1}$ and **assuming informed bidder has not yet dropped out**, we have $n-k$ active bidders that constitute a system of $n-k-1$ inequations for uninformed bidders and a constraint for the informed bidders[16]:

$$\mathbb{E}[V_I|\underline{X}_I^k\leq X_I\leq\overline{X}_I^*]\quad\geq P,\qquad\qquad \text{info}$$

$$\begin{cases}\mathbb{E}[V_1|\underline{X}_I^k\leq X_I\leq\overline{X}_I^*,X_i|_{1\leq i\leq n-k};\underline{X}_j\leq X_j\leq\overline{X}_j|_{n-k+1\leq j\leq n}] & \geq P_k,\\ \vdots & \vdots \qquad \text{uninfo}\\ \mathbb{E}[V_{n-k}|\underline{X}_I^k\leq X_1\leq\overline{X}_I^*,X_i|_{1\leq i\leq n-k};\underline{X}_j\leq X_j\leq\overline{X}_j|_{n-k+1\leq j\leq n}] & \geq P_k,\end{cases}$$

[16]For the equation $\mathbb{E}[V_1|X_1^k\leq X_1\leq\hat{X}_{cut}^*,...,X_{N-k};X_{N-k+1},...,X_N]=P$, it degenerates into $\mathbb{E}[V_1|X_1^k\leq X_1\leq\hat{X}_{cut}^*]\geq P$

where the informed bidder's signal is denoted by X_I, $\underline{X}_I^k$ denotes the insider's the lower bound in round k, $X_1, X_2, ..., X_{n-k}$ indicate remain active bidders' signals, and price P_k is the bidder $n-k$'s last posting price. The upper bound for the insider $\overline{X}_I^*$ depends on whether X_I locates in the first or second cutoff points, i.e., $\overline{X}_I^* = \hat{X}_{1,cut}^*$ when $\underline{X}_I^k \leq \hat{X}_{1,cut}^*$ holds. Otherwise $\overline{X}_I^*$ is relaxed ($\overline{X}_I^* = \infty$). Without loss of generality, we assume that the informed bidder decides whether to drop out or not first. If so, the remaining bidders continue to the next round. If not, we know that informed bidder's signal is no less than P_k, implying $\mathbb{E}[V_I|P_k = \underline{X}_I^k \leq X_I \leq \overline{X}_I^*] \geq P_k$. Hence, the lower bound $\underline{X}_I^k$ is pinned down by P.

Meanwhile, uninformed bidders also update the insider's cutoff points as prices increases. Because the first cutoff point $x_{1,cut}$ depends on the common value components by equation (3), uninformed bidders will leverage their bidding functions to predict the common value part of the item, i.e.,

$$\hat{c}^k = \mathbb{E}[c|x_i|_{1 \leq i \leq n-k}, \underline{x}_I^k \leq x_I \leq \hat{x}_{1,cut}; \underline{x}_j \leq x_j \leq \overline{x}_j|_{n-k+1 \leq j \leq n}],$$

and plug back into equation (3) to infer the first cutoff point $\hat{x}_{1,cut}$. The calculation of expected evaluation of c and corresponding first cutoff point $x_{1,cut}$ can be checked in the appendix. As bidding prices increase, it is possible that the posting price exceeds the first cutoff point $\hat{x}_{1,cut}$. Then the remaining uninformed bidders learn that the informed bidder actually has a very good signal which locates above the second cutoff point $\hat{x}_{2,cut}$. They then bid more cautiously and will stop bidding when the posting price reaches their self expected valuation $\mathbb{E}[V_i|X_i]$. We can get the early stop point from the crossing point of $\mathbb{E}[V_i|X_i]$ and $\mathbb{E}[V_i|X_1, ..., X_N]$.

The existence of the two cutoff points dampens the bidding activities.[17] Heuristically speaking, if

[17] This is compared with the case when uninformed bidders do not consider the first cutoff point. Recall the uninformed bidder's conditional expected valuation given x_{-i}

$$(1-\bar{\beta}_3)p = \bar{\beta}_0^k + \bar{\beta}_1 x_i + \bar{\beta}_2 x_{-i}, i = 2, 3, ..., N-k$$

When there is no cutoff points, uninformed bidders believe the informed bidder's signal satisfy $x > p$ if she does not drops out, resulting in $\int_{\underline{x}_1^k}^{\infty} x_I f(x_I) dx_I$ for the expected signal of the uninformed bidder. When considering the cutoff point, these uninformed bidders' beliefs towards the informed bidder become $\int_{\underline{x}_1^k}^{\hat{x}_{cut}^*} x_I f(x_I) dx_I$, turning the system of equations into

$$p - \int_{\underline{x}_1^k}^{\infty} x_I f(x_I) dx_I < p - \bar{\beta}_3 \int_{\underline{x}_1^k}^{\hat{x}_{cut}^*} x_I f(x_I) dx_I = \bar{\beta}_0^k + \bar{\beta}_1 x_i + \bar{\beta}_2 x_{-i}, i = 2, 3, ..., N-k$$

From the conditional expectation of the informed bidder, it is not hard to infer that $\underline{x}_1^k \geq q$, implying that $\int_{\underline{x}_1^k}^{\infty} x_I f(x_I) dx_I > \int_{\underline{x}_1^k}^{\hat{x}_{cut}^*} x_I f(x_I) dx_I$. Through standard algebra operation, higher left hand side value implies higher signal for the dropout bidder in round k. In other words, bidders with the same signal will drop out early if they perceive the informed bidders cutoff point. This clearly show that having the cutoff point will discourage the bidding activities and reduce the perception of other bidders' signals.

most bidders drop out early, it would imply that the selling item may not be so worthwhile, which reduces the bidding outcomes. On the other hand, if many bidders have relatively higher signals, the common value components would be very high. Naturally the insider should hide her identity when participating the auction. If revealing, It is likely that the insider also has a very high private component. Facing the pressure from the "winner's curse", other bidders will tend to stop early to avoid the unnecessarily loss.

Equilibrium bidding strategies when the insider does not reveal her identity In this scenario, from Assumption 3, uninformed bidders ignore the existence of the informed bidder and follow the bidding strategies as if under symmetric environment. Then we construct the system of inequations in round k:

$$\begin{cases} \mathbb{E}[V_1|X_1;\underline{X}_j \leq X_j \leq \overline{X}_j|_{n-k+1\leq j\leq n}] & \geq P_1^k, \\ \vdots & \vdots \\ \mathbb{E}[V_{n-k}|X_{n-k};\underline{X}_j \leq X_j \leq \overline{X}_j|_{n-k+1\leq j\leq n}] & \geq P_{n-k}^k. \end{cases}$$

Here, everyone act as the same type, bidders no longer have to back out the informed bidder's signal. They will follow the same algorithm to back out the lower and upper bounds. Then bidders will formulate their pivotal bidding function and compete against each other. It is possible that an insider will hide her identity and pretend to be an outsider for the auction. This happens when the insider's signal lies between the first and second cutoff point. If there indeed exists an insider, hiding her identity will be beneficiary otherwise the bidding will be more aggressive. Now, it is time to derive the equilibrium bidding strategies.

3.4 Equilibrium bidding strategies

After characterizing the system of inequations for symmetric and asymmetric cases, we now define the equilibrium bidding strategies. As the auction proceeds to round k, we have the past bidding path $\{P_{t-1},...P_1\}$ along with the corresponding posting prices $\{P_i^k\}_{i=1}^n$. Since the bidding history is common knowledge, bidder i can infer the history dependent lower bounds of private signals $\{\underline{X}_1^k,...,\underline{X}_N^k\}$ by inverting the pivotal bidding functions, i.e., $\underline{X}_j^k = (\beta_j^k)^{-1}(P_j^k;\Omega_k)$, where P_j^k represents the highest bidding price of bidder j at round k. In what follows, we focus on increasing bidding strategies (Hong and Shum, 2003; Haile and Tamer, 2003). Notice that bidders have been sorted by their posting prices such that $i = 1$ indicates the winner. At the beginning of the round k, the lower bounds of the bidders' private signals can be constructed by system equations of N

conditional expectations:

$$
\begin{aligned}
\mathbb{E}[V_1|\underline{X}_1^k,\underline{X}_2^k...,\underline{X}_N^k;\Omega_t] &= P_1^k,\\
\mathbb{E}[V_2|\underline{X}_1^k,\underline{X}_2^k...,\underline{X}_N^k;\Omega_t] &= P_2^k,\\
&\vdots\vdots\\
\mathbb{E}[V_N|\underline{X}_1^k,\underline{X}_2^k...,\underline{X}_N^k;\Omega_t] &= P_N^k,
\end{aligned}
\tag{5}
$$

where $\underline{X}_1^k$, $\underline{X}_2^k,...,\underline{X}_N^k$ are the random variables representing the lower bounds of signals. and P_i^k, $i = 1,...,n$ represents the highest bidding price for bidder i up to the beginning of round k. If bidder i has not submitted the bid, his highest posting price is set to reserve price $P_i^k = \gamma$. The system of equations in (5) is justified and we can get the unique solutions for $\underline{X}_1^k$, $\underline{X}_2^k...,\underline{X}_N^k$, which is summarized in the following lemma.

Lemma 3. *The solution of the n unknown variables in equations.* (5) *are unique and strictly increasing in* $P^k = \{P_1^k,...,P_n^k\}$ *, for all possible realizations of the history dependent lower bounds of* $\underline{X}_1^k$, $\underline{X}_2^k,...,\underline{X}_N^k$.

Lemma 3 relates the existence of a monotonic equilibrium to the nonlinear system of equations (5). With the help of Lemma 3, we can back out the history dependent lower bounds, $\{\underline{X}_1^k, \underline{X}_2^k,...,\underline{X}_N^k\}$, of private signals for each bidder. Regarding the upper bounds, we introduce a high level assumption:

Assumption 5. *when making decisions, bidder expects all her rivals will dropout and she will win in the next period.*

Whenever submitting a new bid, bidders will reveal part of their private signals to the public. A rational bidder will choose to keep the information revelation frequency as low as possible. Thus, a typical bidder chooses to bid only when she must bid, i.e., a situation that she expects all her rivals will dropout. Otherwise, the winner would be the current posting bidder. We apply this assumption to construct the system of equations to recover the upper bounds for the rivials' private signals of bidder i at period t:

$$
\begin{aligned}
\mathbb{E}[V_1|\overline{X}_1^k,\overline{X}_2^k...,\overline{X}_n^k;\Omega_t] &= P_1^k+\Delta,\\
\mathbb{E}[V_i|\overline{X}_1^k,\overline{X}_2^k...,\overline{X}_n^k;\Omega_t] &= P_i^k+\Delta,\\
&\vdots\vdots\\
\mathbb{E}[V_N|\overline{X}_1^k,\overline{X}_2^k...,\overline{X}_n^k;\Omega_t] &= P_n^k+\Delta,
\end{aligned}
\tag{6}
$$

where $\overline{X}_1^k, \overline{X}_2^k ..., \overline{X}_N^k$ excluding $\overline{X}_i^k$ are the unknown upper bounds for bidder i's rivals, and $P_i^k + \Delta$ represents the bidder i's next period bidding price. Similarly, we get the lemma 4.

Lemma 4. *The solution of the n unknown variables in equations* (6) *are unique and strictly increasing in $P_i^k + \Delta$, for all possible realizations of the upper bounds of $\overline{X}_1^k, \overline{X}_2^k ..., \overline{X}_N^k$.*

In the symmetric case, the history dependent upper bounds are equal across bidders, i.e., $\overline{X}_i^k = \overline{X}_j^k,\ i, j \in \{1, 2, ..., N\}$. The following proposition states that the above three assumptions plus the two lemmas are sufficient to ensure the existence of equilibrium bidding strategies, where the proof can be found in the appendix.

Proposition 2. *For auction τ with n number of bidders, given **Assumption** 1, 3, 5 and **Lemma** 3, 4, there exists pivotal bidding functions in round k for uninformed bidder under symmetric case,*

$$\beta_i^k(X_i, \Omega_k) = \mathbb{E}[V_i | X_i, \underline{X}_j^k \leq X_j \leq \overline{X}_j^k,\ \forall j \neq i; \Omega_k], \tag{7}$$

for bidder $i = 1, ...n$, where lower bound, $\underline{X}_j^k$, upper bound, $\overline{X}_j^k$, and public information, Ω_t, are defined above.

When there exists an (revealed) insider, the pivotal bidding function turns into

$$\bar{\beta}_i^k(X_i, \Omega_k) = \begin{cases} \mathbb{E}[V_i | X_i, \underline{X}_I^k \leq X_I \leq \overline{X}_I^*, \underline{X}_j^k \leq X_j \leq \overline{X}_j^k,\ j \neq i, I; \Omega_t] & p \leq \hat{X}_{1,cut}^* \\ \mathbb{E}[V_i | X_i] & p \geq \hat{X}_{2,cut}^* \end{cases},$$

where $\{\hat{X}_{1,cut}^*, \hat{X}_{2,cut}^*\}$ are defined above. Depending on the predicted cutoff points, bidders may stop bidding early intentionally. And the informed bidder follows

$$\bar{\beta}_I^k(X_I, \Omega_k) = X_I.$$

Moreover, there exists a Bayesian Nash Equilibrium in strictly increasing bidding strategies when

$$\begin{cases} \beta_i^k(X_i, \Omega_k) & \geq P_i^k \\ \bar{\beta}_i^k(X_i, \Omega_k) & \geq P_i^k \end{cases},\ i = 1, ..., n-k \text{ or } I, \tag{8}$$

where posting price in round k, P_i^k, and minimal bidding increment, Δ, are defined above.

Unlike the unique bidding strategy in button auction format, **Proposition** 2 defines an equilibrium bidding rule that allows bidders to have multiple bidding strategies, as long as these strategies satisfy the pivotal bidding functions. This provides a feasible way to deal with model incompleteness issue in an English auction model.

Because of log-normal assumption, the system of equations (5), (6) and (7) is log-linear in the signals, allowing us to derive the trackable equilibrium bidding functions for each period. Mathematically speaking, the conditional expectation of V_i take the form:

$$\mathbb{E}[V_i|X_1,...,X_n;\theta,\Omega_t] = \exp\left(\mathbb{E}(v_i|x_1,..,x_n;\theta,\Omega_k) + \frac{1}{2}Var(v_i|x_1,...,x_n;\theta,\Omega_k)\right), i = 1,...,n. \quad (9)$$

Furthermore, we denote the mean and variance–covariance matrix of $(v_i, x_1, ...x_n)$ by $\mu_i \equiv (\mu_i, \mu^*)$ and $\Sigma_i \equiv \begin{bmatrix} \sigma_i^2 & \sigma_i^{*\prime} \\ \sigma_i^* & \Sigma^* \end{bmatrix}$. Then, the conditional mean and variance of jointly normal random variables for the history dependent lower bound private signals are:

$$\mathbb{E}[v_i|\underline{x} \equiv (\underline{x}_1^k,...,\underline{x}_n^k)';\theta,\Omega_k] = (\mu_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^*) + (\underline{x}^k)'\Sigma^{*-1}\sigma_i^*, \quad (10)$$

and

$$Var(v_i|\underline{x}^k;\theta,\Omega_k) = \sigma_i^2 - \sigma_i^{*\prime}\Sigma^{*-1}\sigma_i^*. \quad (11)$$

The linearity of the signals helps us to recover the upper and lower bounds from the system of equations. Finally, we can recover the pivotal bidding function (7) by plugging the lower bound $\underline{x}^k$ and upper bound, $\bar{x}^k$, into the equations:

$$\mathbb{E}[V_i|X_i,\underline{X}_j^k \leq X_j \leq \bar{X}_j^k, j \neq i;\theta,\Omega] = \exp\begin{pmatrix} \mathbb{E}(v_i|x_i,\underline{x}_j^k \leq x_j \leq \bar{x}_j^k, \forall j \neq i;\theta,\Omega)+ \\ \frac{1}{2}Var(v_i|x_i,\underline{x}_j^k \leq x_j \leq \bar{x}_j^k, \forall j \neq i;\theta,\Omega) \end{pmatrix}. \quad (12)$$

If there exists an insider, we transform the upper and lower bounds into the corresponding cutoff points as discussed in the previous subsection.

$$\begin{cases} \mathbb{E}[V_i|X_i,\underline{X}_j^k \leq X_j \leq \overline{X}_j^k,\ \forall j \neq i,I;\underline{X}_I^k \leq X_I \leq \hat{x}_{1,cut}^k;\Omega_k\ ;\theta] \geq P_i^k & \text{lower signal} \\ \mathbb{E}[V_i|X_i,\underline{X}_j^k \leq X_j \leq \overline{X}_j^k,\ \forall j \neq i,I;\hat{x}_{2,cut}^k \leq X_I;\Omega_k\ ;\theta] \geq P_i^k & \text{higher signal} \end{cases} \quad (13)$$

The pivotal function described in equation (12) and (13) can be plugged back into the bidding strategies in the previous subsection. And we finish the construction for the two-stage model with the identity revelation decision of the insider. Based on pivotal function and the identity revelation decision rules established so far, we have several implications summarized in the next subsection.

3.5 Winner's and loser's curse

Based on the model setup, we can further discuss the strategic interactions during the bidding activities and the value of information. The pivotal function further provides the relation between the bidder i's private signal X_i.

Corollary 1. *In pivotal bidding functions, the expected value is increasing with respect to $\underline{X}_j$ and $\overline{X}$. And under the same expected value, i.e. $E = \mathbb{E}[V_i|X_i, \underline{X}_j \le X_j \le \overline{X}, \forall j \neq i, \Omega_t; \theta]$, X_i is non-increasing with respect to $\underline{X}_j$ and $\overline{X}$.*

The proof can be checked by directly expanding the expression of the conditional expectation $\mathbb{E}[V_i|X_i, \underline{X}_j \le X_j \le \overline{X}, \forall j \neq i, \Omega_t; \theta]$. Before moving on, let me reformulate the pivotal function (12) into logarithmic form with the notational convenient settings:

$$\begin{array}{c}\mathbb{E}(v_i|x_i, \underline{x}_{-i} \le x_{-i} \le \bar{x}_{-i}; \theta) + \\ \frac{1}{2}Var(v_i|x_i, \underline{x}_j \le x_j \le \bar{x}_j, \forall j \neq i; \theta)\end{array} = A + \beta_1 x_i + \beta_2 \cdot \mathbb{E}[\underline{x}_{-i} \le x_{-i} \le \bar{x}_{-i}], \quad (14)$$

where A summarizes the constant part; β_1 is the coefficient in front of x_i, while β_2 is the coefficient vectors $(1 \times (N-1))$ in front of the column vector x_{-i}, i.e. $\Sigma_{-i}^{*-1}\sigma_i^*$, and Σ_{-i}^{*-1} is the $(N-1) \times N$ matrix excluding ith row.

Although asymmetric information will make uninformed bidder in a disadvantage bidding situation, but sometimes it can also mitigate the winner's curse. By definition, the winner's curse refers to the fact that bidder i realizes that others have lower private signals:

$$\mathbb{E}[V_i|X_i = X_i^*, X_j < X_i^*, j \neq i; \theta, \Omega] - \mathbb{E}[V_i|X_i = X_i^*; \theta, \Omega].$$

where X_i^*indicates bidder i's private signal. The net difference is negative, meaning that winning brings bad news (Krishna, 2009). Moreover, the larger the x_i or N, the worse the curse will be. On the other side, the common value auction may also induces the loser's curse: early dropout may also be bad news (Pesendorfer and Swinkels, 1997). Similar to the winner's curse, the information premium for the loser's curse can be expressed as:

$$\mathbb{E}[V_i|X_i = X_i^*; \theta, \Omega] - \mathbb{E}[V_i|X_i = X_i^*, X_i^* < X_j,\ j \neq i; \theta, \Omega].$$

As bidder i's private signal X_i^* increases, the marginal premium is increasing as well. The winner's curse and loser's curse are like the mirror image in which the private signal X_i determines the relative strength of the two effects. As the lower bound $\underline{X}_j$ increases, the curse will be alleviated, which is explained in the appendix. If the lower bounds of other bidders are high, the item tends to have higher common value, alleviating the curse of winning the item. Information revelation usually reduces the information premium of the high valued bidder. As the price gradually rises up, a (uninformed) bidder will realize that someone may have a higher signal and start following the high valued bidder's bidding behavior. This will induce more aggressive competition that reduces the premium of winning the auction.

The above discussion of uninformed bidder's equilibrium bidding strategies shows that knowing more information may not be always beneficiary to the information disadvantaged group. The information revelation and learning process in the Common value open ascending auction gives the bidder the possibility to manipulate the bidding outcomes through different strategic behavior. So far, we have characterized the equilibrium bidding behavior for different types of bidders. Based on our theoretical results, we are able to design the identification strategy and construct the structure estimation procedure to recover the parameters that we are interested in.

4 Identification and Structural Estimation

There are two primary challenges in backing out the value distribution from the data. The first is the data unobservability issue. In the Chinese judicial auction mechanism, only one bidder's information is displayed per period, leaving it unclear whether other bidders dropped out earlier or if multiple bids were submitted but only one was shown. This opacity prevents both econometricians and bidders from observing rivals' dropout prices, a key distinction in this auction format.

Another issue comes from partial identification induced by model incompleteness and multi-dimensional information structure (Bikhchandani et al., 2002; Haile and Tamer, 2003; Athey and Haile, 2002). We have multiple equibilira from value of signals to the bidder's bidding strategies, complicating the mapping from model to the data. The two issues make the point estimation infeasible. Instead, we take advantages of the moment inequality conditions to get interval estimation. In this section, we first discuss the identification strategy for each of the parameters in θ. Then, we establish the structure estimation procedure to guide our empirical analysis.

4.1 Identification strategy

To address the model incompleteness problem, Haile and Tamer (2003) construct the lower and upper bounds for each bidder's signals in the auction. Our research takes advantage of this approach under interdependent information structure. Because the bidders face multi-dimensional signals (both private and common value and the existence of an informed bidder), we need to find ways to separate out different component of the signals.

To recover the value distribution, the mean shifter μ_τ is correlated with a list of proxies that indicates the demographic or geographic features of the auction and selling item. Hence, we can control $\mu_\tau(\Omega)$ using the public information of the auction Ω along with demographic characteristics from the data. Then we can get the demeaned bidding prices ready to back out the variance of the distribution.

The key parameters of interest include the variance of the private component σ_a^2, the common value part σ_c^2, and the noisy part σ_ε^2. Since different components affect value distributions in different directions. we leverage within and across auction variation. Moreover, we can utilize the second highest bidder's bidding price to help us narrow down the intervals of parameters (Aradillas-López et al., 2013). Heuristically, from the coefficient vector of bidder's expected valuation, we see that a higher σ_c shifts the expected evaluation upward, while σ_a and σ_ε offset against each other. Particularly, a higher σ_a leads bidders care more about their own signals, making the bidding prices more sparse. On the contrary, a higher σ_ε force bidders care more about rival's signals, especially the insider's. This lead to the concentration of the bidding prices.[18]

First, to identify the common value part variation, we know that bidders within the same auction receive the identical common value shocks. Such variation across auctions, especially for different number of bidders, can help us identify common value component. Moreover, since all bidders within an auction receives the same common value component realization, we can utilize the price

[18] Mathematically speaking,

$$\mathbb{E}(v_i|x_i,\underline{x}^t_{-i} \leq x_{-i} \leq \bar{x}_{-i};\theta) = A + \beta_1 x_i + \mathbb{E}[\underline{x}^t_{-i} \leq x_{-i} \leq \bar{x}_{-i}] \cdot \beta_2$$

where A summarizes the constant part, i.e $(\mu_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^*)$ from (10), β_1 is the coefficient in front of x_i while β_2 is the coefficient vectors $((N-1)\times 1)$ in front of x_{-i}, i.e. $\Sigma^{*-1}_{-i}\sigma_i^*$, where Σ^{*-1}_{-i} is the $(N-1)\times N$ matrix excluding ith row. From matrix algebra, when $\sigma_\varepsilon \uparrow$, it will make all bidders' private signals less precise. Both β_1 and β_2 are negatively affected, resulting in a shrinkage of the signal evaluation, especially for the bidder's own private signals (β_1 drops more severely). Under this scenario, it cause a more concentrated and lower expected value, which further lead to a more concentrated and lower posting bidding price.

when $\sigma_a \uparrow$, it will make the bidder's own private signal more precise and in the mean while, learning effect from other rivals becomes less attractive for bidders. This is because private component dominate the value distribution. In this scenario, β_1 will increase significantly while β_2 decreases more than the case of σ_ε^2 ceteris paribus. In terms of the auction results, we will find a more spread posting bidding price distribution.

difference to net out the common value component to check the distribution difference.

Second, separating the noisy part and private value part is a little harder. One way is to utilize the existence of the informed bidder. In the model, the winning bid variation of the informed bidder only contains the common value part and private value part. By comparing winning bid variation between informed and uninformed bidders, we are able to isolate the noisy part. An alternative way is to compare the range of lower and upper bounds for each bidders. Although the private and noisy parts have the opposite impact on bidders' bidding strategies, a higher private value component can encourage more active bidding activities, while a higher σ_ε could suppress the bidding competition. Regarding this difference, the distance between the lower and upper bounds for each bidder can capture this variation. Hence, we are able to recover the value distribution model by comparing with the (posting) price difference in each auction.

Based on the identification strategy, we derive the following conditions used for the construction of econometric estimation procedure. Using the pivotal functions, we have constructed following system of inequalities:

$$\begin{aligned} \mathring{P}_i \leq \beta(X_i, \Omega_T; \theta) < \mathring{P}_{win} + \Delta \; \forall i \geq 2, \\ \mathring{P}_{win} \leq \beta(X_i, \Omega_T; \theta) \qquad\qquad i = 1, \end{aligned} \tag{15}$$

where $\mathring{P}_i$ indicates the last (highest) bidding price of bidder i , $\Omega_t$indicates information needed up to period T, θ indicates the set of parameters that we care about, and Δ represents the bidding ladder. Particularly, the second highest bidder have the smallest inequality interval, i.e.

$$0 \leq \beta(\overline{X}_2, \Omega_T; \theta) - \beta(\underline{X}_2, \Omega_T; \theta) \leq 2\Delta$$

Remember that inequation (15) implies

$$\underline{x}_j \leq x_j = c + a_j + \varepsilon_j \leq \overline{x}_j, \; j = 1, 2, ... \tag{16}$$

Moreover, we get the lower and upper bounds from these posting prices from the system of equations (5) and (6). Then, we can derive the difference between the posting price to infer the range of signals between different bidders, expressed as

$$0 \leq x_2 - x_j \leq \underline{x}_2 - \underline{x}_j = \underline{x}_2(\mathring{p}_2) - \underline{x}_j(\mathring{p}_j), \; j > 2,$$

which can be reformulated into

$$0 \leq a_2 + \varepsilon_2 - (a_j + \varepsilon_j) \leq \underline{x}_2(\mathring{p}_2) - \underline{x}_j(\mathring{p}_j),\ j > 2. \tag{17}$$

The intuition for the identification is that the set of inequalities illustrate that bidder's real expected evaluation should be close to their last posting bidding price, which is also a stronger version of Assumption 3. Since$\{c, a_j, \varepsilon_j\}$ follows the normal distribution, we can derive the density function and apply maximum simulated likelihood to conduct estimation.

Definition 1. Let Θ_I be such that

$$\Theta_I = \{\theta \in \Theta,\ s.t.\ \text{inequalities (16), (17) are satisfied at } \theta\ \forall X\ a.s.\}.$$

We say that Θ_I is the (sharp) identified set with $\{\sigma_v^2,\ \sigma_\alpha^2,\ \sigma_\varepsilon^2\}$.

4.2 Estimation procedure for maximum simulated likelihood

Based on the identification strategy described in the previous subsection, the estimation procedure can be separated into two part. In the first part, we back out the mean shifter of the value distribution from the data. Then, we can use the demeaned bidding prices to back out the set of parameters that we are interested in, i.e., Θ_I.

In the first step, since the means shift is reflected directly from the bidding prices, we control the mean shifter $\mu_\tau(\Omega)$ using public information of the auction Ω along with demographic characteristics from the data. Specifically, we construct the following econometric equation

$$p_{i,\tau,s,t} = \alpha + \lambda_s + \mu_t + \delta\Omega_{\tau,s,t} + \varepsilon_{i,\tau,s,t}$$

where α indicate the common constant term, $p_{i,\tau,s,t}$ indicate the last post price of bidder i in auction τ at city s and year t. λ_s indicate the city fixed effects, μ_t the year fixed effect, $\Omega_{\tau,s,t}$ indicate the public information and demographic characteristics at the auction level. The resulting residual $\varepsilon_{i,\tau,s,t}$ represent the demeaned posted prices. The first stage regression helps to back out the demeaned price distribution. Then we can focus on the structure estimation for the set of parameters Θ_I.

To recover the set of parameter, the top priority is to pin down the lower and upper bounds of each bidder's signal in auctions. With the help of our parametric model framework, we build the linkage from model to the data to help us recover the lower and upper bounds. However, the multiple equilbria for the bidding strategies during the dynamic bidding activities can shuffle the lower and upper bounds randomly. Instead of finding the lower (upper) envelope of the lower (upper) bounds, which can rise serious equilibrium solving problem, we search for the sup and inf for the range of the bounds per bidder's signal. Specifically, in each equilibrium bidding scenario, we derive the lower and upper bounds from the data. Heuristically, a proper set of parameters should generate the bidder's real signal within the lower and upper bounds with highest possibility. Based on this intuition, we construct the likelihood probability function for the inf and sup of the bound range for estimation. Since recovering the lower and upper bounds needs to integrate the truncated expected signals for the winner, which requires the simulation method for truncated normal distribution., we seek simulated method to reduce the computational burden. Hence, using Maximum simulated likelihood method. The algorithm is sketched as follows:

1. Given an initial set of parameters $\{\sigma_v^s,\ \sigma_\alpha^s,\ \sigma_\varepsilon^s\}$, we back out the lower and upper bounds from the bidding history for each bidder in a given auction.
2. Construct the likelihood function and calculate the likelihood function and compare the value of the likelihood function with the previous results.
3. Search for the parameter space to find the maximum level and the corresponding set of parameters.
4. Repeat above steps until reaching the whole potential parameter space.

After this, we construct the likelihood function (part 1) from cumulative distribution function (CDF), i.e.,

$$LL_1 = \prod_{\tau} \prod_{i=2}^{N_\tau} (F_x(\overline{x}^i) - F_x(\underline{x}_i))$$

where F_x denotes the CDF for x. Recall that for an informed bidder $x_I = c + a_I$. Moreover, we denote $g_{a+e}^{(j)}$ and $G_{a+e}^{(j)}$ as the PDF and CDF of range $a_2 + \varepsilon_2 - (a_j + \varepsilon_j)$. From the property of the order statistics, denote $R = a_2 + \varepsilon_2 - (a_j + \varepsilon_j)$ we have

$$g_{a+e}^{(j)}(r) = n! \frac{1}{(i-1)!} \frac{1}{(n-2-i)!} \int f(z) f(z+r)[1-F(z+r)] F(z)^{i-1} [F(z+r) - F(z)]^{n-2-i} dz$$

If there exists an informed bidder, we make $\varepsilon_I = 0$ and adjust the $g_{a+e}^{(j)}(r)$ accordingly. The variation of R help us to separate out the common value component from the private component, becoming the second part of the likelihood function

$$LL_2 = \prod_{\tau} \prod_{j>2}^{N_\tau} \int_0^{\underline{x}_2(\mathring{p}_2) - \underline{x}_j(\mathring{p}_j)} g_{a+e}^{(j)}(r) dr.$$

Combining the two likelihood functions, the objective function can be defined as

$$
\begin{aligned}
Q(\theta) = \max_{\theta}\{\log LL_1 + \log LL_2\} \\
s.t. \\
\{\bar{x}, \underline{x}\} \text{ from (6) and (5)} \\
\mathring{P}_{win} \leq \beta(X_i, \Omega_T; \theta)
\end{aligned}
$$

The corresponding sample analog is defined as:

$$
\begin{aligned}
Q_n(\theta) = \max_{\theta}\{\log \overline{LL_1} + \log \overline{LL_2}\} \\
s.t. \\
\{\bar{x}, \underline{x}\} \text{ from (6) and (5)} \\
\mathring{P}_{win} \leq \beta(X_i, \Omega_T; \theta)
\end{aligned}
$$

where $\overline{LL_1}$ and $\overline{LL_2}$ are the simulated likelihood function. Moreover, by shifting the bidder's belief towards opponents, we can search for the lowest and highest bounds of the signal. Therefore, the set of the parameters can be pinned down.

5 Empirical Results

I organize the discussion of the results in two steps, First, we present and discuss the results for the value distribution using the Chinese Judicial auction data. Next, to get the sense of information asymmetry for the Judicial auction, we the investigate the information premium using the estimated parameters for the welfare analysis. Following the (Ciliberto and Tamer, 2009; Chernozhukov et al., 2007), I report the confidence region of $\{\sigma_c,\ \sigma_a,\ \sigma_\varepsilon\}$ that is defined as the set that contains the parameters that cannot be rejected as the truth with at least 95% probability.

The main estimation results of the value distribution are shown in Table 5. I estimate parameters of the value distribution under different situations: Column 1 lists the results where I exclude all the priority bidders; Column 2 shows the results for the whole data sample; and Column 3 displays the subsample since 2017. The range is shown in the second row represented by square brackets. And the first row indicates the center point of the estimated parameters. From the Table 5, we see the structure parameters under different sub-samples are consistent. In other worlds, the information asymmetry problem is universal. Regarding the identification power, the results shows the good sensitivity of the private and noisy part for the value distribution. A salient feature of the estimation is the dominant size of noisy component over bidder's private and common value part. The large portion of the noisy part over the bidder's signal distribution shows the importance of the learning process among bidders in this typical auction activities. This also justify that the magnificent influence of the informed bidder to the rest of uninformed bidders' bidding behavior.

The estimation results indicate a possible way for the informed bidders to affect the bidding outcomes by strategically manipulating the uninformed bidder's information structure. From the model, we know that If common value or noisy component is large, uninformed bidders will put much higher weight on the bid submitted by the informed bidder. Therefore, the asymmetric information structure between informed and uninformed bidders gives the informed agent a space for altering the outcome and exploit information premium. We will see more clearly in the following analysis for the value of information.

5.1 Information premium under asymmetric information structure

measuring the distribution and the allocation of information is one of the main goals in this research. In this subsection, we first define the value of information between two types of bidders. Here we provide three different proxies measuring the information premium from different aspects.

First , let ϖ_i^1 be the net changes for expected evaluation of uninformed bidders over the existence of the informed bidder (Dionne et al., 2009):

$$
\begin{aligned}
\varpi_i^1 = &\log(\mathbb{E}_t[V_i|X_i, \underline{X}_I \leq X_I \leq \overline{X}_I,\ \underline{X}_j \leq X_j \leq \overline{X}_j, \forall j \neq i, \Im,\ \Omega_t; \theta]) \\
&- \log(\mathbb{E}_t[V_i|X_i, \underline{X}_j \leq X_j \leq \overline{X}_j, \forall j \neq i,\ \Omega_t; \theta])\ . \qquad (18)
\end{aligned}
$$

The first part in (18) indicates the (logarithmic) conditional expectation of the uninformed bidder i with an insider I as one of his rivals. While the second part indicates the same situation without the informed bidder. The lemma from the appendix, shows how the coefficients varies under the symmetric and asymmetric environment. i.e., $\bar{\beta}_I > \beta_I$, $\bar{\beta}_1 > \beta_1$, and $\bar{\beta}_2 > \beta_2$.

How ϖ_i^1 is correlated with the number of bidders? Under the parametric assumption, we find that the information premium defined in the proposition is *decreasing* with the number of participants N_τ. Heuristically, as more and more bidders flood into the auction, single bidder's influence declines. Although the presence of an informed bidder can induce more weights shifting towards the informed rivals, such effect will be offset by the increasing number of participants. Formally, we can have the following proposition for the information premium:

Proposition 3. *The information premium* ϖ_i^1 *defined by* (18) *is decreasing with the number of participants* N_τ *(*$N_\tau \geq 3$*). As* $N_\tau \to \infty$*,* $\varpi^* \to 0$*. Moreover, considering the expected valuation, we have* $\bar{\beta}_I = 1 - [\bar{\beta}_1 + \bar{\beta}_I + \bar{\beta}_2 \cdot 1_{N_\tau - 2}]$*. Moreover,* $\partial \beta_2 / \partial N_\tau < 0$*,* $\partial \bar{\beta}_I / \partial N_\tau < 0$*, and* $\partial \bar{\beta}_2 / \partial N_\tau < 0$*. In the limit, we have* $\lim_{N_\tau \to \infty} \bar{\beta}_I = 0$ *and* $\lim_{N_\tau \to \infty} \beta_1 + \beta_2 \cdot 1_{N_\tau - 1} = 1$.

The proof is in the appendix. Notice that the bidder i's own private effect is independent with the number of rivals, i.e., $\sigma_a^2/(\sigma_a^2 + \sigma_\varepsilon^2)$. This ratio also indicates the value of the bidder's private signals. The learning process helps to reduce the wedge of $1 - \sigma_a^2/(\sigma_a^2 + \sigma_\varepsilon^2)$. As $N_\tau \to \infty$, we can eliminate the side influence from the noisy part. In other word, the maximum effort for the learning effect coefficients is to fill up $\sigma_\varepsilon^2/(\sigma_a^2 + \sigma_\varepsilon^2)$.

This proxy shows the conventional information premium calculation that uninformed bidders do not consider why the informed bidder is willing to reveal her identity. To include the informed bidder's information revelation decision, we further construct the following two proxies.

Second, let ϖ_i^2 be the net changes for expected evaluation of uninformed bidders over the existence of the informed bidder. Particularly, the uninformed bidders learn that the informed bidder voluntarily make revelation decisions. Hence, they know the informed bidder's signal could be either below the first cutoff point or above the second cutoff point.

$$\varpi_i^2 = \log \bar{\beta}(X_i, \Omega_k) - \log(\mathbb{E}_t[V_i | X_i, \underline{X}_j \leq X_j \leq \overline{X}_j, \forall j \neq i,\ \Omega_t; \theta]) \tag{19}$$

where

$$\bar{\beta}(X_i, \Omega_k) = \begin{cases} \mathbb{E}[V_i | X_i, \underline{X}_I^k \leq X_I \leq \overline{X}_I^*, \underline{X}_j^k \leq X_j \leq \overline{X}_j^k,\ j \neq i, I; \Omega_t] & \underline{X}_I^k \leq \hat{X}_{1,cut}^* \\ \mathbb{E}[V_i | X_i] & \underline{X}_I^k \geq \hat{X}_{2,cut}^* \end{cases}, \tag{20}$$

Correspondingly, we use ϖ_i^3 to measure information premium of those tend to hide identity, i.e., forcing the hidden informed bidder to reveal her identity.

$$\varpi_i^2 = \log \bar{\beta}(X_i, \Omega_k) - \log(\mathbb{E}_t[V_i | X_i, \underline{X}_j \leq X_j \leq \overline{X}_j, \forall j \neq i,\ \Omega_t; \theta])$$

where

$$\bar{\beta}(X_i, \Omega_k) = \mathbb{E}[V_i | X_i, \max\{\underline{X}_I^k, \hat{X}_{1,cut}^*\} \leq X_I \leq \min\{\overline{X}_I^*, \hat{X}_{2,cut}^*\}, \underline{X}_j^k \leq X_j \leq \overline{X}_j^k,\ j \neq i, I; \Omega_t]$$

Compared to ϖ_i^1, ϖ_i^2 and ϖ_i^3 include the value of identity revelation for informed bidders. Based on the estimated value distribution, we can conduct the simulation and counterfactual analysis to understand how the information premium is affected by different factors.

In the simulated results of ϖ_i^1 (Figure 7), we see that as the lower bound of the informed bidder increases, bidder i's updated expected value is rising quickly. but when the updating process only comes from the uninformed bidder, the marginal increment of information value is decreasing, which is consistent with the mathematical expression. This is because bidder i may be worried that the higher bidding price of the uninformed bidders results from a noisy or private signal rather than a higher common value component. In addition, Figure 8 shows the fact that the increasing number of bidders will attenuate the informed bidder's influence.

—

Using the estimated parameters[19], the simulation results are shown in figure 11. As predicted from the model, the presence of the informed bidder will induce more aggressive bidding, resulting in a higher winning bid and more frequent bidding submission. These graphs, the winning bid, the deviation of the bidding price distance, and the biding frequency help to illustrate the influence of the learning process in an open ascending auction.

In general, if the bidders can learn from each other, given other things equal, higher learning effect will induce lower distance. However, when an informed bidder enters the auction, two more effects will be introduced. Because the presence of the informed bidder brings a higher valued signal to the item, the low valued bidders realize that it is not possible for them to win the item. They will stop bidding quickly. This is the screening effect. This can be checked by the left fat tail for the distribution of bidding outcomes with informed bidder. While for those high valued bidders, the presence of informed bidder will give them more confidence to bid aggressively, resulting in a higher winning bid. This is the value revelation effect. Therefore, we see a fat tail under the informed bidder and the left shifted density crest under the informed bidder from the winning price distribution. In general, the presence of the informed bidder add extra volatility to the bidding activities.

6 Analysis for the Bidders' Strategic Behavior

After backing out the estimated parameters, I can utilize the counterfactual analysis to investigate how informational asymmetry affects the bidding behavior and value distribution. Especially, I examine how the presence of the informed bidder affect bidding behavior of other participants. I also investigate how the information premium varies with bidders' bidding activities. Since the informed bidder's bidding strategy may affect the auction outcome, the counterfactual analysis helps to study

[19] In this simulation experiment, I restrict the number of bidders to 5 and separately generate the set of bidding path with respect to the existence of informed bidder.

such strategic interactions.

6.1 Simulation environment setup

I generate the simulated auction data under the model assumptions. Regarding the value distribution parameters, I choose the corresponding parameters from the whole data sample and reach the minimal objective function value. The bidding ladder, Δ, is fixed as the 0.5% of the evaluation value. while the reservation price is normalized to 0.7 of the item's evaluation value.

In the simulated auction, at the very beginning, each bidder only knows their own private signal x_i. Once someone starts to bid, bidders can obtain more and more price information from the bidding process. They will combine their own private signals and others' signals recovered from the bidding activity to update their conditional expectation. After simulating S auctions, we can calculate the moment conditions and distributions that we are interested in.

6.2 Comparison to sealed bid auction

Another way to compare the effect of learning process is to analyze how the bidding outcome would be if the auction is transformed into the sealed bid second price auction. The result is shown in Figure 12. From the graph, we see that by shutting down the learning process, the average winning price increases in the sealed bid auction. However, in an open ascending auction, the high valued item can be sold in a even higher price compared due to the learning process. In other words, because of the information revelation and learning process, an open ascending auction has more volatile bidding outcomes. Some auctions may end with unexpected low or high winning price. In contrast, in the ideal sealed bid auction, the distribution of the winning price is close to the evaluation price.

Such comparison illustrate a key feature of the learning process: the amplification effect. In an open ascending auction, when the private signal can not provide the accurate evaluation of the bidding item, which is usually the case in an common value environment, bidders rely heavily on their rivials' actions. If all the participants bid cautiously at the beginning, the information revealed to the public implies that this selling item probably not worth so much. After learning this, bidders will adjust their strategies and bid even less actively, dampen the competition. On the other side, if some one really like the selling item, the active bidding of this particular bidder will give others the impression that this item could have higher value than they expected, which induce an over reaction on this bidding competition through the learning process. In short, if the selling item on average can provide detailed information indicating the high common value among bidders, the learning process in an open ascending auction will help the auctioneers to earn higher expected value compared with sealed bid auction. However, if the information friction is serious, the expected revenue of the auctioneer will be hard to predict. And under such a scenario, probably a sealed bid auction is a better choice.

6.3 Auction outcomes and the informed bidder's bidding strategy

Regarding the strategic behavior of the informed bidder, I further control the level of bidding activity for the informed bidder under different situations. The experiment procedure is described in the appendix. The counterfactual analysis illustrates three scenarios. First, in normal case, the auctioneer randomly picks one of the bid submitted by all the candidate bidders (including the informed bidder's bid). Second, in active case, the auctioneer deliberately chooses the bid submitted by the informed bidder every other period. Third, in inactive case, the auctioneer randomly picks one of the bid submitted by all but the informed bidder. The experiment results are shown in Figure 13.

From the figure, we see that under the inactive case, the wining price distribution has significantly left shift pattern, which implies by strategically freezing their bidding activities, the informed bidders would have an big impact on the bidding outcomes. Such correlation is sensitive to the component of noisy part, common value part, and private value part in the bidder's value distribution. If noisy part is not very large, which means bidders on average learn the value of the item well. Or the common value part is not very large, which means bidder's private value dominate the value of the item. In both cases, the reliance of other bidder's bidding behavior is dampened. A key implication of this experiment shows getting more information is not always good for agents to improve their welfare or surplus. Sometimes, restricting the insider's behavior could be beneficial to the public.

Another interesting case is the active bidding strategy. Usually, an active informed bidders will induce more competitive bidding environment, the learning effect is salient. However, active bidding behavior also prevent information revealing process of other participants, which drag down the real evaluation of the selling item. Which effect dominates depends on the parameters as well as the number of bidders. If the active informed bidders can perfectly block other bidders' bidding behavior, the more the bidders, the stronger the second effect.

7 Conclusion

In common value English auction, recovering the value distribution is non-trivial because mapping from the value distribution to the bidding distribution is no longer one to one. Due to the multiple equilibria issue and incompleteness of econometric structure, we face the many to many correspondence issue. Often times, bidders have different levels of information towards the selling items. The interactions among different types of bidders directly affect the final outcomes.

This paper aims to address those issues using Chinese Judicial auction data. To solve the multiple equilibria problem, the paper builds a structural auction model that utilizes the bidding history to construct the upper and lower bound for the value distribution. From the estimation result, the large portion of noisy component reduces the auction revenue. Moreover, it gives the informed bidder's leverage to manipulate the bidding outcomes through some strategic bidding behavior. More generally, this paper contributes to the literature that study the information asymmetry.

Through the simulation test, informed bidder can significant affect the auction outcome by strategically choosing her bidding behavior. Indeed, the model is just simple abstraction from the real world that aims to capture the key influence of information structure. More details are waited for further research.

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Appendix A.

A.1 Comparison of coefficient vectors between symmetric and asymmetric information environment

For the convenience of mathematical expression, we first discuss the coefficient vectors of bidder's conditional expected value, i.e., $\Sigma_{\text{sym}}^{*-1} \cdot \sigma_i^*$and $\Sigma_{\text{asy}}^{*-1} \cdot \bar{\sigma}_i^*$.

The Σ_{sym}^* is symmetric and positive definite, we can do the eigenvalue decomposition, where

$$\Sigma_{\text{sym}}^* = \begin{bmatrix} \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 & \sigma_c^2 & \cdots & \sigma_c^2 \\ \sigma_c^2 & \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 & \cdots & \sigma_c^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_c^2 & \sigma_c^2 & \cdots & \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 \end{bmatrix}$$

Based on eigenvalue decomposition, we can first derive the eigenvalue:

$$\begin{aligned} |\Sigma_{\text{sym}}^* - \lambda I| &= \begin{vmatrix} n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2-\lambda & \sigma_c^2 & \cdots & \sigma_c^2 \\ n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2-\lambda & \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 & \cdots & \sigma_c^2 \\ \vdots & \vdots & \ddots & \vdots \\ n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2-\lambda & \sigma_c^2 & \cdots & \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 \end{vmatrix}, \\ &= \begin{vmatrix} 1 & 0 & \cdots & 0 \\ 1 & \sigma_a^2+\sigma_\varepsilon^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & \sigma_a^2+\sigma_\varepsilon^2 \end{vmatrix} (n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2-\lambda), \\ &= (n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2-\lambda)[\sigma_a^2+\sigma_\varepsilon^2-\lambda]^{n-1} = 0. \end{aligned}$$

So we get the eigen-values $\lambda^* = n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2$ and $\lambda_0 = \sigma_a^2+\sigma_\varepsilon^2$. After a series of algebra operation, when multiplying the $\sigma_i^* = \begin{bmatrix} \sigma_c^2, & \sigma_c^2+\sigma_a^2, & \cdots, & \sigma_c^2 \end{bmatrix}^T$, we can get $(\Sigma_{\text{sym}}^*)^{-1}\sigma_i^*$ as :

$$\frac{1}{n}\begin{bmatrix} \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \\ \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}+\frac{n-1\sigma_a^2}{\lambda_0} \\ \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \\ \vdots \\ \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \end{bmatrix} = \underbrace{\frac{1}{n}\begin{bmatrix} \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \\ \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \\ \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \\ \vdots \\ \frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0} \end{bmatrix}}_{\text{learning effect}} + \underbrace{\begin{bmatrix} 0 \\ \frac{\sigma_a^2}{\lambda_0} \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{\text{private valuation}} \tag{21}$$

From equation (21)[20], we can divide the coefficient vector into two terms, the first term varies according to the number of total bidders while the second term is independent of the number of bidders. Therefore, the first term is named as the learning effect, and the second term indicates the bidder's own private value, named as the private valuation effect. The above procedure introduces how bidders learn from each other under the symmetric information environment. The more participants, the more information revelation to the public.

Now we turn to the case that an informed bidder joins in the auction. In this scenario, the information structure has been changed, with $\Sigma_{\text{asy}}^{*-1}$ becomes

$$\Sigma_{\text{asy}}^{*-1} = \begin{bmatrix} \sigma_c^2+\sigma_a^2 & \sigma_c^2 & \cdots & \sigma_c^2 \\ \sigma_c^2 & \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 & \cdots & \sigma_c^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_c^2 & \sigma_c^2 & \cdots & \sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2 \end{bmatrix}$$

Again, from the knowledge of linear algebra, we get

$$\Sigma_I^{*-1}\cdot\sigma_i^* = \left(\sigma_c^2 \begin{bmatrix} \frac{\sigma_\varepsilon^2}{D} \\ \frac{\sigma_a^2}{D} \\ \frac{\sigma_a^2}{D} \\ \vdots \\ \frac{\sigma_a^2}{D} \end{bmatrix} + \sigma_a^2 \begin{bmatrix} 0 \\ \frac{\sigma_a^2+\sigma_c^2}{(n-1)D} \\ \frac{\sigma_a^2+\sigma_c^2}{(n-1)D} \\ \vdots \\ \frac{\sigma_a^2+\sigma_c^2}{(n-1)D} \end{bmatrix} - \sigma_a^2 \begin{bmatrix} 0 \\ \frac{1}{\sigma_a^2+\sigma_\varepsilon^2}\frac{1}{n-1} \\ \frac{1}{\sigma_a^2+\sigma_\varepsilon^2}\frac{1}{n-1} \\ \vdots \\ \frac{1}{\sigma_a^2+\sigma_\varepsilon^2}\frac{1}{n-1} \end{bmatrix} \right) + \begin{bmatrix} 0 \\ \frac{\sigma_a^2}{\sigma_a^2+\sigma_\varepsilon^2} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{22}$$

where $D = \sigma_a^4 + \sigma_a^2(n\sigma_c^2+\sigma_\varepsilon^2) + \sigma_c^2\sigma_\varepsilon^2$. From the equation 22, the coefficients are grouped by different effects, the first term indicates the learning effect, the second term indicates the information

[20] Under the algebra simplification, $\frac{1}{n}[\frac{n\sigma_c^2}{\lambda^*}+\frac{\sigma_a^2}{\lambda^*}-\frac{\sigma_a^2}{\lambda_0}] = \frac{\sigma_\varepsilon^2\sigma_c^2}{\lambda^*\lambda_0}$

hidden effect from the informed bidder, and the last term indicates the bidder's own private effect à la the uninformed case.

A.2 Information revelation decision: Proof of Lemma 1

This part discusses information revelation decision for the insider. Before the auction begins, the insider first receives her signal and then decide whether to reveal her identity or not. For the convenience of the model deduction, we assume that first, the insider has the highest signal, i.e., the potential winner of the item; second, bidders drop out at their expected valuation price. Later on, we will relax the second assumption for generalization.

When making the revelation decision, the informed bidder will consider the reaction of a typical uninformed bidder under different situations. If the informed bidder decides not to reveal the identity, the uninformed bidder 2, without loss of generality, will treat all his rivals have the same information structure. The updating rule is

$$\mathbb{E}[v_i|x_2,x_i,i\in\{3,4,...,N\},x_1^I]+\frac{1}{2}Var(v_i|x_i,x_{-i};\theta,\Omega_t)),$$

where the second term is of conditional variance is constant by the property of normal distribution. Notice that x_1^I represent the informed bidder's signal but not revealing her identity. Since bidder 2 believe the environment is symmetric, we have the following expression (as in equation (10)):

$$\begin{aligned}\mathbb{E}[v_i|x_2,x_i,i\in\{3,4,...,N\},x_1^I] &= (\mu_i-\mu^{*\prime}\Sigma^{*-1}\sigma_i^*)+(x_i,x_{-i})'\Sigma^{*-1}\sigma_i^* \\ &= A+\beta_1x_2+\beta_2\sum_{i=3}^{N}x_i+\beta_3x_{1,cut}^I \end{aligned} \tag{23}$$

where $A=(\mu_i-\mu^{*\prime}\Sigma^{*-1}\sigma_i^*)+\frac{1}{2}\left[\sigma_i^2-\sigma_i^{*\prime}\Sigma^{*-1}\sigma_i^*\right]$ represent the constant component, β_1, β_2 and β_3 are the coefficients from the decomposition of $\Sigma_{sym}^{*-1}\sigma_i^*$. Based on the decomposition of coefficient vectors $\Sigma^{*-1}\sigma_i^*$, we can see that $\beta_2=\beta_3$, $\beta_1>\beta_2$, and $\beta_1-\beta_2=\frac{\sigma_a^2}{\lambda_0}$. On the other hand, if the informed bidder decides to reveal the identity, The updating rule for bidder 2 becomes:

$$\begin{aligned}\bar{\mathbb{E}}[v_i|x_2,x_i,i\in\{3,4,...,N\},x_1^I] &= (\mu_i-\mu^{*\prime}\Sigma^{*-1}\sigma_i^*)+(x_i,x_{-i})'\Sigma^{*-1}\sigma_i^* \\ &= \bar{A}+\bar{\beta}_1x_2+\bar{\beta}_2\sum_{i=3}^{N}x_i+\bar{\beta}_3x_{1,cut}^I \end{aligned} \tag{24}$$

where $\bar{A}$ is the constant term, $\bar{\beta}_1$, $\bar{\beta}_2$ and $\bar{\beta}_3$ are the coefficients from the decomposition of $\Sigma_{asy}^{*-1}\sigma_i^*$.

Next, we will investigate how the two functions varies with the informed bidder's signal conditional on the uninformed bidders' signals. The first step is to find the relationship between the two sets of coefficient vectors. Therefore we have the following lemma.

Lemma 5. *From the uninformed bidder's updating rule and the information structure under two different environments. We have that* $\bar{\beta}_3 > \beta_3$, $\bar{\beta}_2 < \beta_2$, *and* $\bar{\beta}_1 < \beta_1$.

Recall that coefficient vectors for asymmetric and symmetric cases $\Sigma_{\text{asy}}^{*-1} \cdot \sigma_i^*$ and $\Sigma_{\text{sym}}^{*-1} \sigma_i^*$ are

$$\Sigma_{\text{asy}}^{*-1} \cdot \sigma_i^* = \left(\sigma_c^2 \begin{bmatrix} \frac{\sigma_\varepsilon^2}{D} \\ \frac{\sigma_a^2}{D} \\ \frac{\sigma_a^2}{D} \\ \vdots \\ \frac{\sigma_a^2}{D} \end{bmatrix} + \sigma_a^2 \begin{bmatrix} 0 \\ \frac{\sigma_a^2+\sigma_c^2}{(n-1)D} \\ \frac{\sigma_a^2+\sigma_c^2}{(n-1)D} \\ \vdots \\ \frac{\sigma_a^2+\sigma_c^2}{(n-1)D} \end{bmatrix} - \sigma_a^2 \begin{bmatrix} 0 \\ \frac{1}{\sigma_a^2+\sigma_\varepsilon^2}\frac{1}{n-1} \\ \frac{1}{\sigma_a^2+\sigma_\varepsilon^2}\frac{1}{n-1} \\ \vdots \\ \frac{1}{\sigma_a^2+\sigma_\varepsilon^2}\frac{1}{n-1} \end{bmatrix} \right) + \begin{bmatrix} 0 \\ \frac{\sigma_a^2}{\sigma_a^2+\sigma_\varepsilon^2} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\Sigma_{\text{sym}}^{*-1} \sigma_i^* = \frac{1}{n} \begin{bmatrix} \frac{n\sigma_c^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \\ \frac{n\sigma_c^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \\ \frac{n\sigma_c^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \\ \vdots \\ \frac{n\sigma_c^2}{\lambda^*} + \frac{\sigma_a^2}{\lambda^*} - \frac{\sigma_a^2}{\lambda_0} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\sigma_a^2}{\lambda_0} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

where $\lambda^* = n\sigma_c^2 + \sigma_a^2 + \sigma_\varepsilon^2$, $\lambda_0 = \sigma_a^2 + \sigma_\varepsilon^2$, and $D = \sigma_a^4 + \sigma_a^2(n\sigma_c^2 + \sigma_\varepsilon^2) + \sigma_c^2\sigma_\varepsilon^2$. And the symmetric learning effect per element is $\sigma_\varepsilon^2 \left[\frac{\sigma_c^2}{(\sigma_a^2+\sigma_\varepsilon^2)(n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2)} \right]$. While the learning effects from the informed x_1^I and uninformed rivals x_i are

$$\begin{cases} \frac{\sigma_\varepsilon^2\sigma_c^2}{D} & \text{informed} \\ \frac{\sigma_c^2\sigma_a^2(n-1)+\sigma_a^2(\sigma_a^2+\sigma_c^2)}{D(n-1)} - \frac{1}{(n-1)}\frac{\sigma_a^2}{\sigma_a^2+\sigma_\varepsilon^2} = \frac{\sigma_a^2\sigma_\varepsilon^2\sigma_c^2}{D(\sigma_a^2+\sigma_\varepsilon^2)} & \text{uninformed} \end{cases}$$

Therefore $\bar{\beta}_3 - \beta_3$ become:

$$\begin{aligned} \bar{\beta}_3 - \beta_3 &= \frac{\sigma_\varepsilon^2\sigma_c^2}{\sigma_a^4 + \sigma_a^2(n\sigma_c^2 + \sigma_\varepsilon^2) + \sigma_c^2\sigma_\varepsilon^2} - \sigma_\varepsilon^2 \left[\frac{\sigma_c^2}{(\sigma_a^2 + \sigma_\varepsilon^2)(n\sigma_c^2 + \sigma_a^2 + \sigma_\varepsilon^2)} \right] \\ &= \sigma_\varepsilon^2\sigma_c^2 \frac{1}{(\sigma_a^4 + \sigma_a^2(n\sigma_c^2 + \sigma_\varepsilon^2) + \sigma_c^2\sigma_\varepsilon^2)(\sigma_a^2 + \sigma_\varepsilon^2)(n\sigma_c^2 + \sigma_a^2 + \sigma_\varepsilon^2)} \sigma_\varepsilon^2 \left[(n-1)\sigma_c^2 + \sigma_a^2 + \sigma_\varepsilon^2 \right] \\ &> 0 \end{aligned}$$

Similarly, it is straightforward to see that $\bar{\beta}_2 - \beta_2 < 0$:

$$\bar{\beta}_2 - \beta_2 = \frac{\sigma_a^2\sigma_\varepsilon^2\sigma_c^2}{D(\sigma_a^2+\sigma_\varepsilon^2)} - \left[\frac{\sigma_\varepsilon^2\sigma_c^2}{(\sigma_a^2+\sigma_\varepsilon^2)(n\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2)}\right] = -\frac{\sigma_c^2\sigma_\varepsilon^2}{1}\frac{\sigma_c^2\sigma_\varepsilon^2}{D\lambda^*\lambda_0}.$$

For the constant part, we need to compare the summation of $1\Sigma_{\text{asy}}^{*-1}\cdot\sigma_i^*$ and $1\Sigma_{\text{sym}}^{*-1}\sigma_i^*$. Notice that the private value part is identical for the symmetric and asymmetric case. It is the learning effect term that determines the relationship between $\bar{A}$ and A. From $\bar{\beta}_3 - \beta_3 > 0$, the first row in $1[\Sigma_{\text{asy}}^{*-1}\cdot\sigma_i^* - \Sigma_{\text{sym}}^{*-1}\sigma_i^*]$ is positive. But the rest of rows are negative, (e.g.,$\bar{\beta}_2 - \beta_2 < 0$). But the summation is positive since $(\bar{\beta}_3-\beta_3)+(n-2)(\bar{\beta}_2-\beta_2)+(\bar{\beta}_1-\beta_1) \equiv (\bar{\beta}_3-\beta_3)+(n-2)(\bar{\beta}_2-\beta_2)+(\bar{\beta}_1-\beta_1)$

$$\begin{aligned}
1[\Sigma_{\text{asy}}^{*-1}\cdot\sigma_i^* - \Sigma_{\text{sym}}^{*-1}\sigma_i^*] &= \sigma_\varepsilon^2\sigma_c^2\frac{1}{D\lambda^*\lambda_0}\sigma_\varepsilon^2\left[(n-1)\sigma_c^2+\sigma_a^2+\sigma_\varepsilon^2\right] - (n-1)\frac{\sigma_c^2\sigma_\varepsilon^2}{1}\frac{\sigma_c^2\sigma_\varepsilon^2}{D\lambda^*\lambda_0} \\
&= \frac{\sigma_\varepsilon^4\sigma_c^2}{D\lambda^*\lambda_0}\left[\sigma_a^2+\sigma_\varepsilon^2\right] > 0
\end{aligned}$$

So considering the constant term $A = u_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^* + \frac{1}{2}\left[\sigma_i^2 - \sigma_i^{*\prime}\Sigma^{*-1}\sigma_i^*\right]$, we get that $A_{sym} > A_{asy}$ holds. Moreover, fixing other signals $x_2, \ldots x_n$, we see that the expected value of bidder 2 is increasing with respect to x_1^I. The slop is β_2 and $\bar{\beta}_2$.

Lemma 6. *Given the information structure, the constant term in uninformed bidder's conditional expectation is defined as*

$$A_f = u_i - \mu^{*\prime}\Sigma_f^{*-1}\sigma_i^* + \frac{1}{2}\left[\sigma_i^2 - \sigma_i^{*\prime}\Sigma_f^{*-1}\sigma_i^*\right],\ f \in \{sym, asy\}.$$

Moreover, $A_{sym} > A_{asy}$.

So far, we have shown that uninformed bidder's expected updating rules, (23) and (24), have the crossing point. But the point not only depends on x_1^I, but also other bidders' signals. To find the crossing point, the informed bidder needs to integrate out other bidders' signals, i.e., $x_2, \ldots, x_n$. The reason that a typical uninformed bidder cares rivals' signal is because the common value component from the item value. Hence, the cutoff point is closely correlated with the common value realization c. To see this, assuming the common value part of the item is c. Conditional on c, the informed bidder will derive the expected updating rules so as to pin down the cutoff point. Depending on her own private value a_I, the informed bidder will decide whether to reveal her identity or not.

Based on the information structure, we know that conditional on c, $x_2, \ldots x_n$ are i.i.d. distributed. To get the typical (the second highest) uninformed bidder's expected valuation, we take advantage of the property of order statistics. Using the expectation of the order statistics, the informed bidder now can construct the bidding decisions with respect to the updating rules under symmetric case:

$$\mathbb{E}\left[A+\beta_1 x_2+\beta_2\sum_{i=3}^{n} x_i+\beta_3 x_1^I|c\right]=L(c)+\beta_1\mathring{\mathbb{E}}(1,N-1)+\beta_2\sum_{i=3}^{N}\mathring{\mathbb{E}}(i-1,N-1)+\beta_3 a_I$$
$$where$$
$$L(c)=A+c[\beta_1+(N-2)\beta_2+\beta_3]$$

and asymmetric case

$$\mathbb{E}\left[\bar{A}+\bar{\beta}_1 x_2+\bar{\beta}_2\sum_{i=3}^{N} x_i+\bar{\beta}_3 x_1^I|c\right]=L(c)+\bar{\beta}_1\mathring{\mathbb{E}}(1,N-1)+\bar{\beta}_2\sum_{i=3}^{N}\mathring{\mathbb{E}}(i-1,N-1)+\bar{\beta}_3 a_I$$
$$where$$
$$L(c)=\bar{A}+c[\bar{\beta}_1+(N-2)\bar{\beta}_2+\bar{\beta}_3]$$

Since the order statistic of a sequence of random variables y_1, y_2, ..., y_n is just a rearrangement of the random variables, the equation $\sum_{i=1}^{n} y_{(i)}=\sum_{i=1}^{n} y_i$ holds in all generality. Based on this property, we know that

$$\sum_{i=3}^{N}\mathring{\mathbb{E}}(i-1,N-1)=\sum_{i=2}^{N}\mathbb{E}(y)-\mathring{\mathbb{E}}(1,N-1)=0-\mathring{\mathbb{E}}(1,N-1). \tag{25}$$

Now we define the decision function h and the corresponding cutoff point x_{cut}

$$\begin{aligned}
h(x_1^I|c,N)&=\mathbb{E}\left[A+\beta_1 x_2+\beta_2\sum_{i=3}^{N} x_i+\beta_3 x_1^I|c\right]-\mathbb{E}\left[\bar{A}+\bar{\beta}_1 x_2+\bar{\beta}_2\sum_{i=3}^{N} x_i+\bar{\beta}_3 x_1^I|c\right]\\
&=A-\bar{A}+c[\Delta\beta_1+(N-2)\Delta\beta_2+\Delta\beta_3]\\
&+\Delta\beta_1\mathring{\mathbb{E}}(1,N-1)-\Delta\beta_2\mathring{\mathbb{E}}(1,N-1)+\Delta\beta_3 a_I\\
&=A-\bar{A}+c[\Delta\beta_1+(N-2)\Delta\beta_2+\Delta\beta_3]+\Delta\beta_3 a_I\\
&=A-\bar{A}+c[\Delta\beta_1+(N-2)\Delta\beta_2]+\Delta\beta_3 x_1^I
\end{aligned}$$

Notice that with the help of (25), $\Delta\beta_1\mathring{\mathbb{E}}(1,N-1)-\Delta\beta_2\mathring{\mathbb{E}}(1,N-1)$ are canceled out ($\Delta\beta_1=\Delta\beta_2$). And we know that $\Delta\beta_1>0$ and $\Delta\beta_2>0$. Therefore, the higher c (assuming $c>0$), the lower the constant term, resulting in a smaller private value component a_I, $\frac{da_{cut}}{dc}>0$. Meanwhile, we also learn that the relationship between a_I and c: the higher c is, the smaller a_I will be. The above results prove the Proposition 1. If we allow early dropout, we will over estimate lower ranked bidders' signals i.e., $\sum_{i=3}^{N}\mathbb{E}[x_i|x_j\geq \underline{x}_j\forall j]\geq\sum_{i=3}^{N}\mathbb{E}[x_i]$. This will add an extra negative constant term denoted by ΔE

on the left hand side of $h(x_1^I|c,N)$.

We define $c = g(x_{cut})$:

$$c = -\frac{A - \bar{A} + \Delta E}{[\Delta\beta_1 + (N-2)\Delta\beta_2]} + \frac{-\Delta\beta_3}{[\Delta\beta_1 + (N-2)\Delta\beta_2]} x_1^I$$
$$= \gamma_0 + \gamma_1 x_{cut}$$

It is not hard to get $\gamma_1 > 1$. Notice that the validation of this information revelation argument depends on an implicit assumption that the common value part is low $c < 0$. When $c < 0$, we have $x_{cut} > c$ and $a_I > 0$. This justifies our key points that when the selling item is high valued, the informed bidder does not have any incentive to reveal the identity. Only when the selling item is not so valuable, the informed bidder can leverage her information advantage to earn extra premium. Revealing the identity will further reinforce other bidders' negative expectation towards the common value part, dampening the competition.

A.3 Proof of Proposition 2

Similar to the proof of Proposition 2 in Hong and Shum (2003), we show that if all bidders $j \neq i$ follow their own equilibrium strategies $\beta_j^k(\cdot)$ in round k, bidder i's best response is to play $\beta_i^k(\cdot)$ because this guarantees that bidder i will win the auction if and only if her expected net payoff is positive conditional on winning.

Given the current bidding price of bidder i, P_i^k, if bidder i wins the auction at the beginning of round k when all remaining bidders simultaneously exit at a price of P_i^k, her ex-post valuation is:

$$\mathbb{E}[V_i | X_i, \underline{X}_j^k \leq X_j \leq \overline{X},\ \forall j \neq i; \Omega_k].$$

Since this conditional expectation is increasing in X_i (from Assumption 1), bidder i makes a positive expected profit from winning in period t by staying actively in the auction at a price of P if and only if $X_i \geq (\beta_i^k)^{-1}(P_i^k; \Omega_t) \Leftrightarrow \beta_i^k(X_i; \Omega_t) \geq P_i^k$. Typically, $\beta_i^k(X_i, \Omega_t)$ specifies the price below which bidder i makes a positive expected profit by staying in the auction and above which bidder i makes a negative expected profit by staying in the auction. Therefore, for every realization of X_i, $\beta_i^k(X_i; \Omega_t)$ specifies a best-response dropout price for bidder i in round k.

A.6 Proof for the Proposition 3

The key to the proof is to derive the analytical expression of $\Sigma^{*-1}\sigma_i^*$ under different situation, i.e., with or without the informed bidder. For simplicity I will put the informed bidder, if there exist one, in the first position and the bidder i in the second position ($i = 2$, $\mathfrak{I} = 1$). The inverse matrix for uninformed and informed cases is denoted by Σ^*_{sym} and Σ^*_{asy}, respectively.

First, under the symmetric case of $\Sigma^{*-1}_{\text{sym}}\sigma_i^*$ in equation (21), it is straightforward to see that as the number of participants increases, the side impact from the noisy component of the bidder's signal will be addressed by the learning effect of numerous rivals. Mathematical speaking,

$$\lim_{n\to\infty} 1 \cdot (\Sigma_0^*)^{-1}\Gamma = 1.$$

Second, under the asymmetric case of $\Sigma^{*-1}_{\text{asy}}\bar{\sigma}_i^*$ in equation (22), we see that in the first term, the weighting from the insider (the first row) is closely related to the noisy component. Also, the presence of the informed bidder will prevent the bidder learning from other informed rivals' behavior, reducing the learning effects from other uninformed bidders. This also explains why informed bidders can manipulate the bidding outcomes through his/her strategic bidding behavior. But such influence will be offset by the increasing number of participants. Since more and more people join in the auction, the behavior of the informed bidder becomes not so valuable any more. We get the following relationship:

$$1 \cdot (\Sigma_0^*)^{-1}\Gamma = 1 - \frac{2\sigma_c^2\sigma_a^2 + \sigma_c^2\sigma_\varepsilon^2}{D}$$

where $\frac{2\sigma_c^2\sigma_a^2+\sigma_c^2\sigma_\varepsilon^2}{D}$ is equivalent to the informed bidder's learning coefficient. As $n \to \infty$, $\lim_{n\to\infty} 1 \cdot (\Sigma_0^*)^{-1}\Gamma = \lim_{n\to\infty} 1 - \frac{2\sigma_c^2\sigma_a^2+\sigma_c^2\sigma_\varepsilon^2}{D} = 1$ as well.

A.4 Explanation of Curse Relief.

The conjuncture is equivalent to show as $\underline{X}$ increases,

$$\frac{d}{d\underline{X}}\left\{\mathbb{E}[V_i|X_i; x_i \geq X_j \geq \underline{X}_j,\ \forall j \neq i] - \mathbb{E}[V_i|X_i; X_j \geq \underline{X}_j,\ \forall j \neq i]\right\} < 0.$$

Due to the property of the conditional expectation and truncated normal distribution, we learn that

$$\frac{d}{d\underline{X}}\left\{\mathbb{E}[V_i|X_i;x_i \geq X_j \geq \underline{X}_j,\ j \neq i] - \mathbb{E}[V_i|X_i;X_j \geq \underline{X}_j,\ \forall j \neq i]\right\} \ltimes \frac{d}{d\underline{X}}R\left\{\frac{\phi(\underline{X}_j) - \phi(x_i)}{\Phi(x_i) - \Phi(\underline{X}_j)} - \frac{\phi(\underline{X}_j)}{1-\Phi(\underline{X}_j)},\ \forall j \neq i\right\}$$

Without loss of generality, I assume that $\alpha = \underline{X}_j$ and $\beta = x_i$. Then we can get

$$\frac{\phi(\alpha)-\phi(\beta)}{\Phi(\beta)-\Phi(\alpha)} - \frac{\phi(\alpha)}{1-\Phi(\alpha)} = \frac{\phi(\alpha)[1-\Phi(\beta)] - \phi(\beta)[1-\Phi(\alpha)]}{(1-\Phi(\alpha))(\Phi(\beta)-\Phi(\alpha))} \equiv A(\alpha)$$

It is not hard to see that

$$\frac{d}{d\alpha}A(\alpha) = \frac{\phi(\alpha)[-\alpha(1-\Phi(\beta)) + \phi(\beta) + (1-\Phi(\alpha))]}{(1-\Phi(\alpha))^2(\Phi(\beta)-\Phi(\alpha))^2},$$

where $\phi'(\alpha) = -\alpha\phi(\alpha)$. As $\alpha\uparrow$, $-\alpha(1-\Phi(\beta)) + \phi(\beta) + (1-\Phi(\alpha)) < 0$, which indicates the alleviation of the curse.

Appendix B. Tables and Graphs

Table 5: Estimated results for value distribution

	No priority bidder	whole sample	After 2017
σ_c	0.2023	0.2032	0.211
	[0.1558, 2460]	[0.1530, 0.2535]	[0.1658, 0.2760]
σ_a	0.1335	0.1328	0.1382
	[0.1035, 0.160]	[0.1018, 0.1628]	[0.1080, 0.1550
σ_ε	0.251	0.2474	0.2547
	[0.2160, 0.3064	[0.2270. 0.3075]	[0.2064, 0.3154]
μ			
Reserve price	0.9971	0.9971	0.9971
Has priority bidder	0.0395	0.0395	0.0395
Min Obj Value	0.1118	0.1125	0.1272
N	8223	8294	6495

Table 6: Information Parmeter for Simulation

name	moment	parameter	value
common value c	mean	μ_c	-0.35
	std	σ_c	0.2023
private value a	mean	μ_a	0
	std	σ_a	0.1382
noise ε	mean	μ_ε	0
	std	σ_ε	0.2474
control coefficient r			0.7

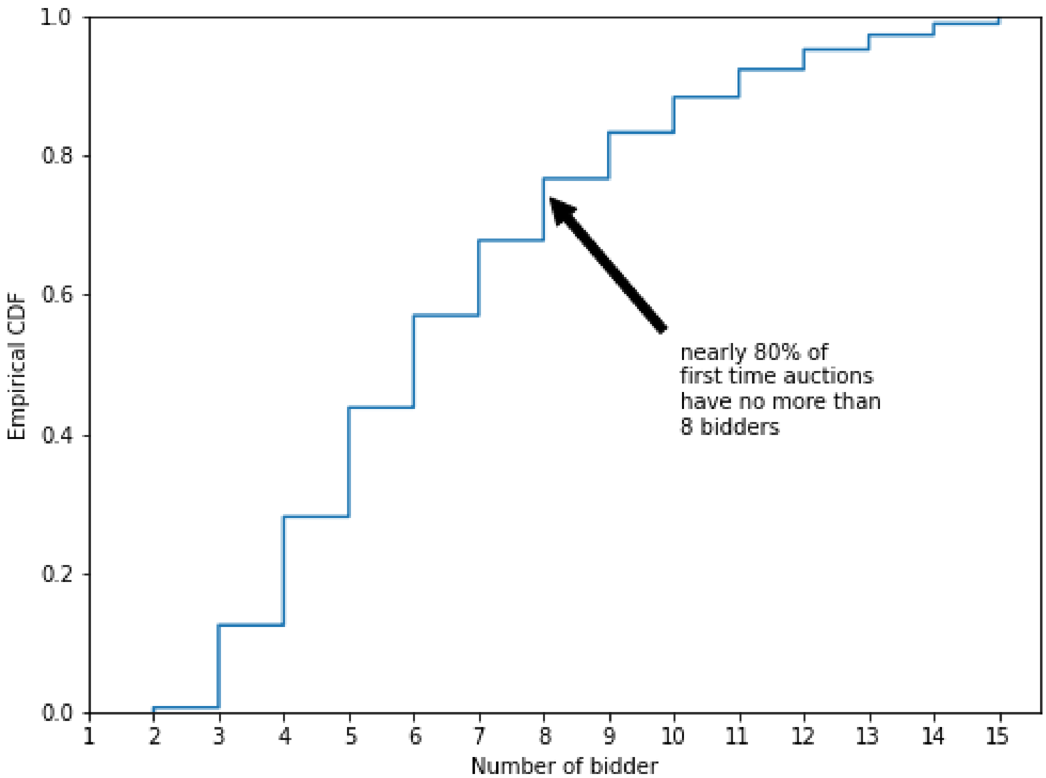
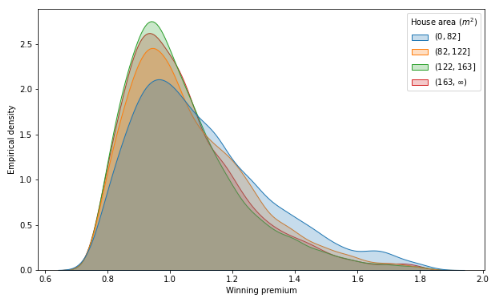
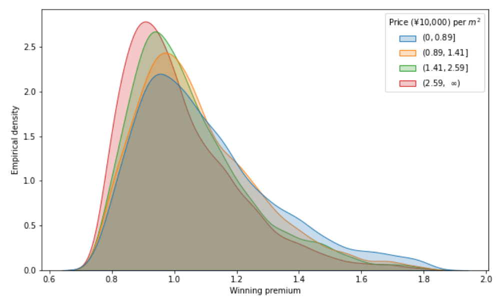


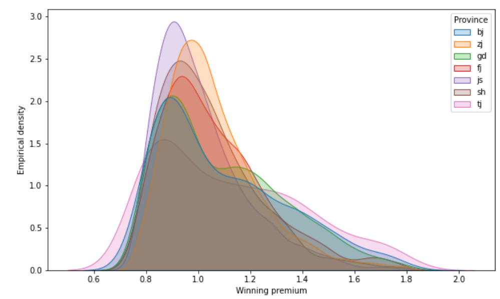
Figure 4: Empirical Distribution for the number of bidders
This figure shows the empirical distribution for the number of bidders participated in each auction. Here I restrict the number of bidders no higher than 15 (Only less than 0.5% of auctions have more than 15 bidders).



(a) House area



(b) Evaluated price per square meters



(c) Different Geographic Locations

Figure 5: The relation between winning premium and different features in the auction

These graphs display the heterogeneous distribution of Judicial Auction winning premium under different classifications. The first one is classified by the house area, the second graph is based on prices per square meters, and the third one is classified by the province level geographic locations. The winning premium is defined as before: the winning price over the evaluation price.

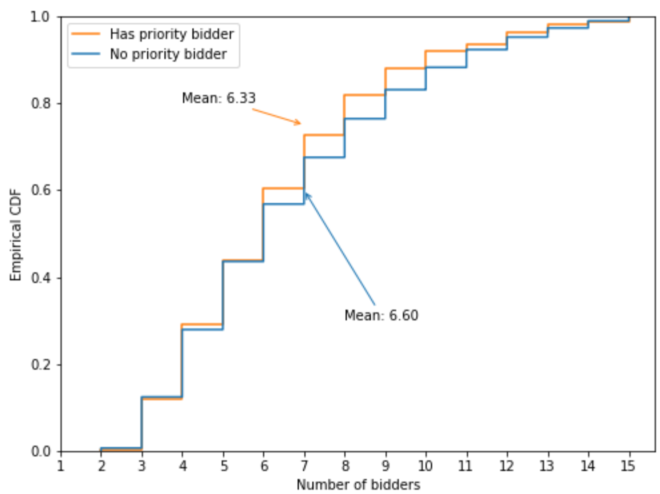
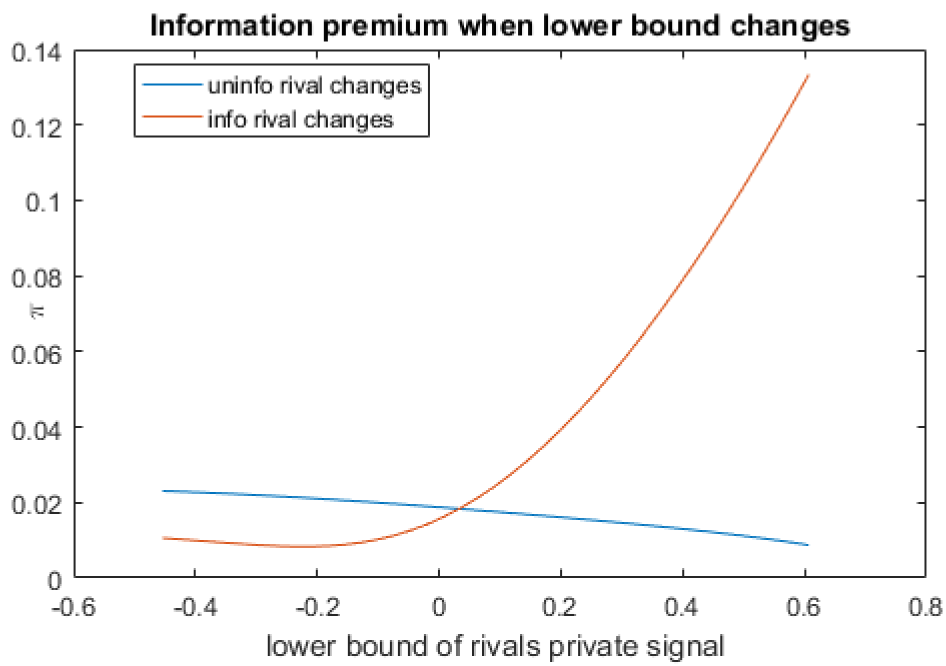


Figure 6: Number of bidders for the priority bidder

This figure shows that auctions with priority bidders on average have less participants. The mean value of the bidders under auctions with a priority bidder is 6.33 while the number is 6.60 under auctions without a priority bidder.

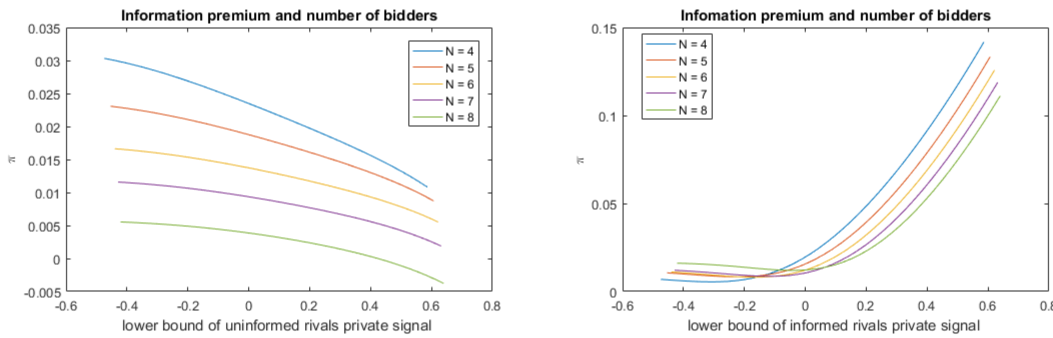
Figure 7: Information Premium and Variations of lower bounds



In this numerical example, I fix the number of bidders equal to 5 and change the lower bound for an informed and uninformed rivals. The parameter set is assumed to be $\mu_x = -0.06$, $r = 0.7$, $\sigma_c = 0.2$, $\sigma_a = 0.15$, $\sigma_\varepsilon = 0.25$, $\Delta = 0.005$. And the variation for the rival's lower bound ranges from -0.4539 to 0.6068. The value is picked from the lower bound recovered from the logarithm of the reservation ratio plus three standard distance from the $\sigma_x = \sqrt{\sigma_c^2 + \sigma_a^2 + \sigma_\varepsilon^2}$. The unchanged rival's lower bound is the middle point of the variation, 0.0764. And the bidder i's private signal is assigned with the mean value -0.06.

The graph indicates how the changes of information premium respond to the lower bound of rival's private signal. The blue line indicates the case when the uninformed rival raises the lower bound, which has a downward pattern. And the red line indicates the case when the informed rival changes the lower bound, indicating an increasing pattern. This implies that a high valued informed rival will significantly incentivize bidders to have a higher evaluation and bid more aggressively.

Figure 8: Information Premium and Number of bidders



Similar to Figure 7, I change the number of bidders to see how the premium varies as the number of bidders increases. The parameter set is assumed to be $\mu_x = -0.06$, $r = 0.7$, $\sigma_c = 0.2$, $\sigma_a = 0.15$, $\sigma_\varepsilon = 0.25$, $\Delta = 0.005$. The number of bidders ranges from 4 to 8. The variation for the rival's lower bound ranges from -0.4539 to 0.6068. The value is picked from the lower bound recovered from the logarithm of the reservation ratio plus three standard distance from the $\sigma_x = \sqrt{\sigma_c^2 + \sigma_a^2 + \sigma_\varepsilon^2}$. The unchanged rival's lower bound is the middle point of the variation, 0.0764. And the bidder i's private signal is assigned with the mean value -0.06.

The left side figure displays the case for the changes of the uninformed rival, while the right side figure depicts the informed rival's case. Consistent with the proposition conclusion, As the number of bidders increases, the information premium decreases quickly, even to the negative value.

Figure 9: Winning outcome distribution under the informed bidder's strategic behavior

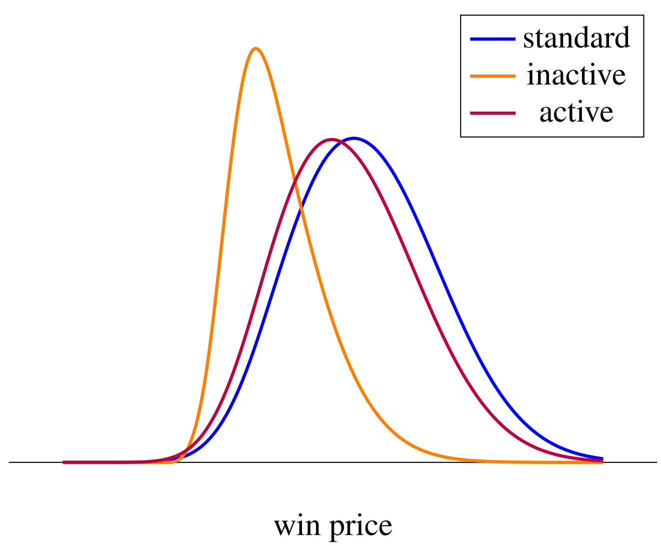
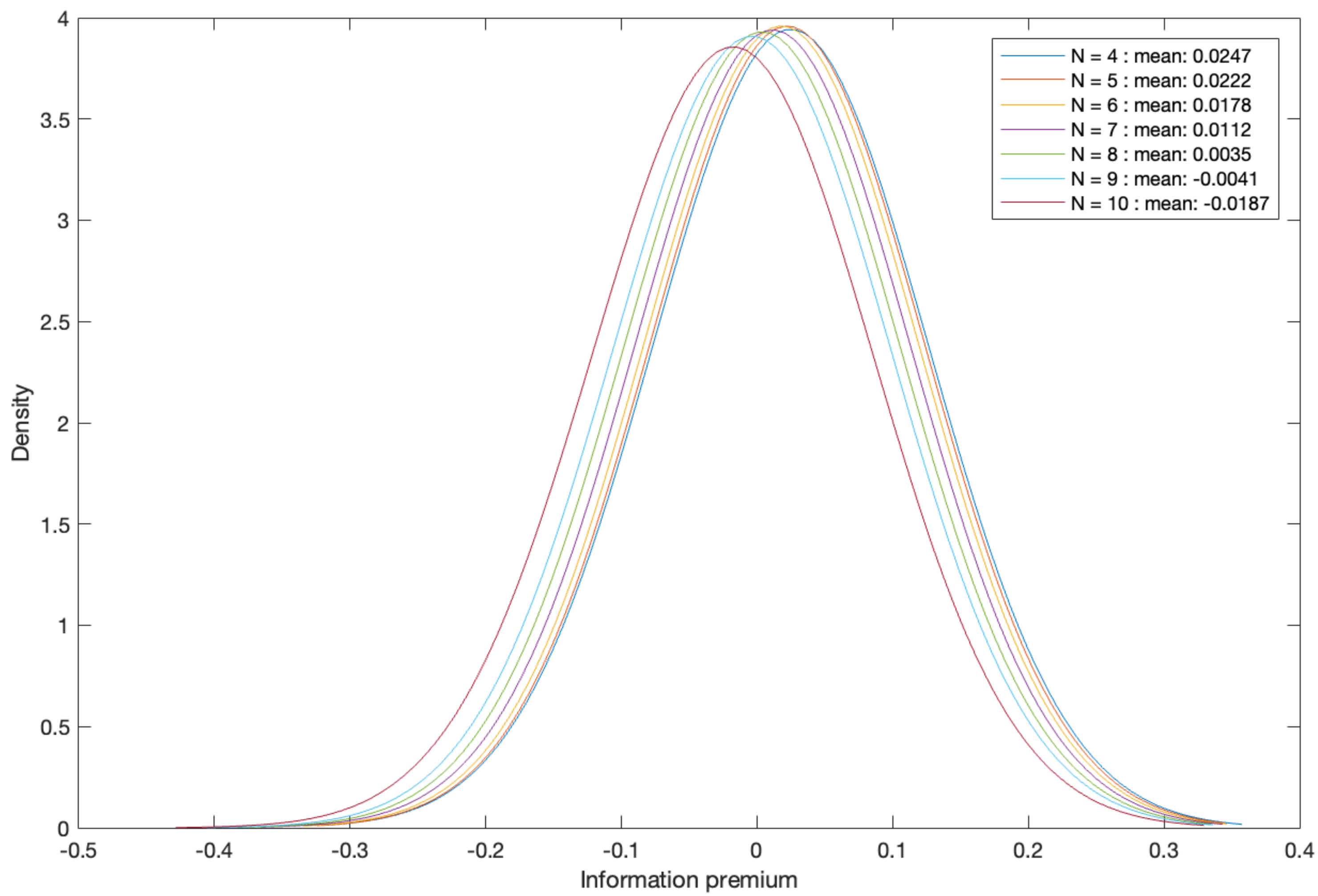
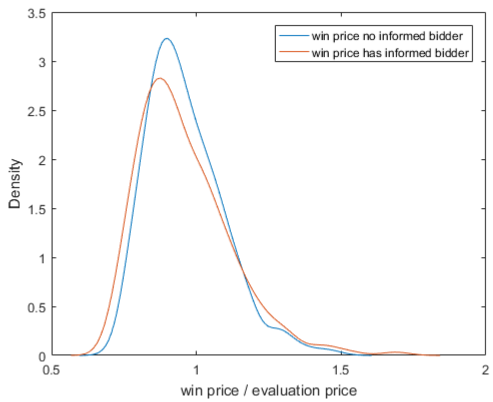


Figure 10: Information premium varies with the number of bidders

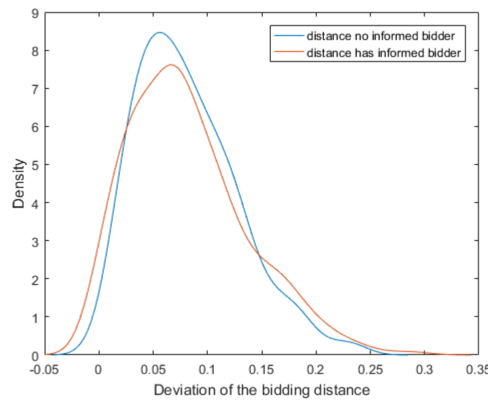


This figure is draw by the smoothing kernel density. The simulation test is selected from 4 to 10 number of bidders with the same base parameter set: $\{\sigma_v = 0.2,\ \sigma_a = 0.15,\ \sigma_\varepsilon = 0.25\}$ with $\mu = -0.06$ and bidding ladder 0.002.

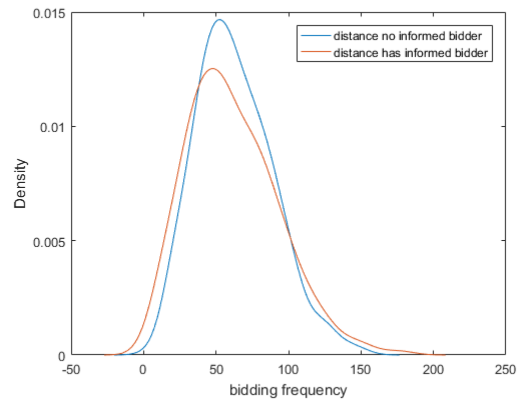
Figure 11: Simulation Results



(a) Winning bid



(b) Deviation of bidding distance



(c) Bidding frequency

The three graphs here show that the main statistic results (distributions) for the simulated results. Figure 11a shows the distribution of winning premium of auctions with and without the priority bidder. Similarly, Figure 11b shows the bidding spread and Figure 11c displays the total bidding length of the auction. On average, auctions with the priority bidder has the fat tail.

Figure 12: Comparison between open and sealed bid auctions

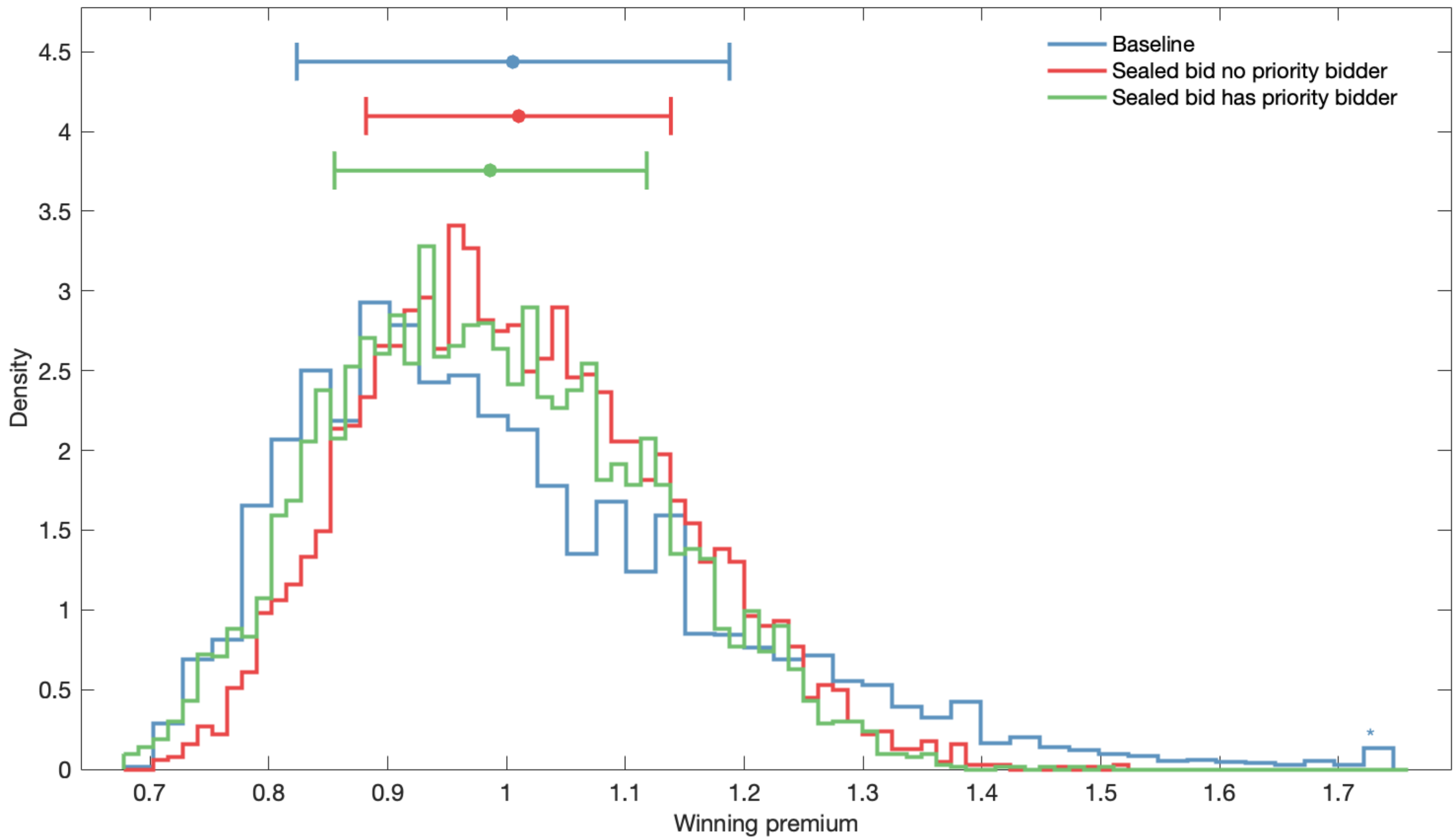


Figure 13: Counterfactual analysis of priority bidder's behavior